Three Concepts: Information

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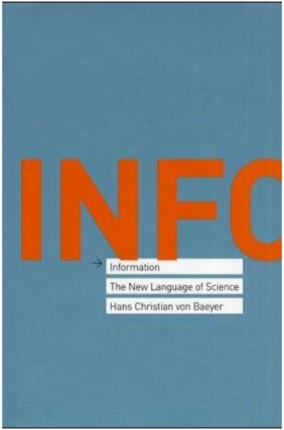
Fall 2007



"Whether on the internet, encoded in radio waves or coursing through wires, information is all around us. Our senses record it, our brains process it and our genes pass it on. But what exactly is information? Can it be analysed and measured? In this extraordinary book, Hans Christian von Baeyer illuminates a concept that could soon become as central to science as space, time mass or energy."

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   - Course details
   - Prerequisites
   - What do I need to do?
   - Grading

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   - Codes
   - Compression
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3. What we will *not* talk about (except today)
   - *Reliable* communication
   - Error correcting codes
   - Repetition codes
   - Shannon's theorem

581286, Three Concepts: Information

- An advanced studies (laudatur) course.

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- `www.cs.helsinki.fi/group/cosco/Teaching/Information/2007/`

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No formal prerequisites **but** you will need

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- Calculus: integrals, derivatives, convergence, ...

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Prerequisites

No formal prerequisites **but** you will need

- Calculus: integrals, derivatives, convergence, ...
- Probability theory: joint & conditional distributions, expectations, law of large numbers, ...
- Programming: language is up to you (but need to work in groups).
- (Verbal) communication: individual and group presentations and written reports.

What do I need to do?

No regular exercises. Instead:

- Home assignments.

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- Projects.

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You do *not* have to attend the classes, unless otherwise stated. However, we recommend that you do.

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No regular exercises. Instead:

- Home assignments.
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- Term paper.

You do *not* have to attend the classes, unless otherwise stated. However, we recommend that you do.

If you find that the course is not for you, please let us know *before* the project groups are formed. After that you are expected to stick with your group.

Grading

The course grading is based on:

1. Projects (50 %)

Grading

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Late policy: 10 % of the available points are reduced for being late and for every 24h that your solution is late.

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Coding Game

Form groups of 3–4 persons. Each group constructs a *code* for the letters A–Z by using as *code-words* unique sequences of dots • and dashes (—) like "•", "— •", " — • — —", etc.

A	________	G	________	M	________	S	________	Y	________
B	________	H	________	N	________	T	________	Z	________
C	________	I	________	O	________	U	________		
D	________	J	________	P	________	V	________		
E	________	K	________	Q	________	W	________		
F	________	L	________	R	________	X	________		

Coding Game

Use your code to *encode* the message
"`WHAT DOES THIS HAVE TO DO WITH INFORMATION`".

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- A dot •: 1 units.
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• • • — — — • • •: $1 + 1 + 1 + 2 + 2 + 2 + 1 + 1 + 1 = 10$.

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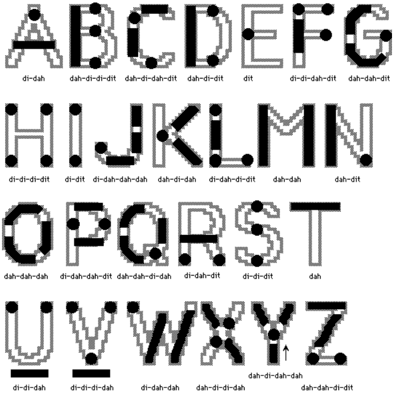
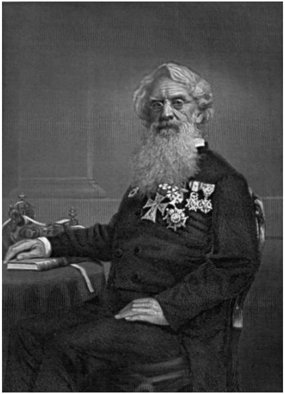
$\bullet\bullet\bullet$ — — — $\bullet\bullet\bullet$: $1+1+1+2+2+2+1+1+1=10$.

The *coding rate* of your code is the length of the encoded message divided by the length of the original message, including spaces (42).

Coding Game





Samuel F.M. Morse (1791–1872)

Coding Game

WHAT DOES THIS HAVE TO DO WITH INFORMATION

Coding Game

```
WHAT DOES THIS HAVE TO DO WITH INFORMATION

.-- .... .- -   -.. --- . ...   - .... .. ...
.... .- ...- .   - ---   -.. ---   .-- .. - ....
.. -. ..-. --- .-. -- .- - .. --- -.
```

Coding Game

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WHAT DOES THIS HAVE TO DO WITH INFORMATION
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```
.-- .... .- -   -.. --- . ...   - .... .. ...
.... .- ...- .   - ---   -.. ---   .-- .. - ....
.. -. ..-. --- .-. -- .- - .. --- -.
```

51 dots, 36 dashes, 7 spaces: $51 + 72 + 14 = 137$ units.

Coding Game

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WHAT DOES THIS HAVE TO DO WITH INFORMATION
```

```
.-- .... .- -   -.. --- . ...   - .... .. ...
.... .- ...- .   - ---   -.. ---   .-- .. - ....
.. -. ..-. --- .-. -- .- - .. --- -.
```

51 dots, 36 dashes, 7 spaces: $51 + 72 + 14 = 137$ units.

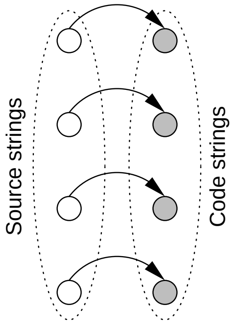
Morse code

Coding rate: $\dfrac{137}{42} \approx 3.26$

Did you do better or worse? Why?

Codes

Lossless compression:
injective mapping



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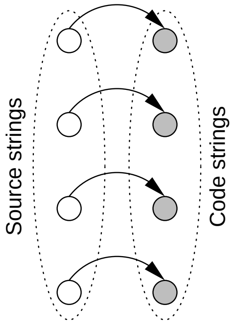
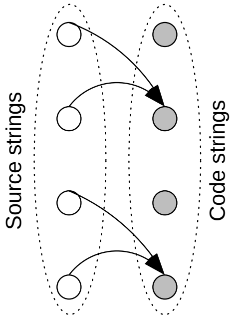


Lossy compression:
non-injective mapping



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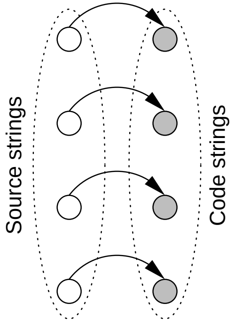

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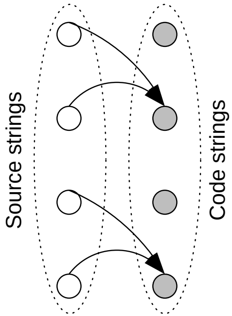

Only *lossless* codes are *uniquely decodable*.

Codes

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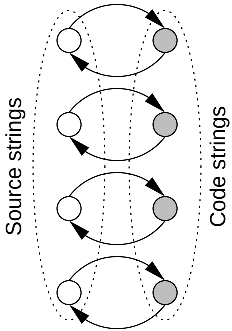

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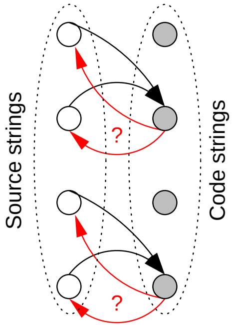

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Examples

general purpose

`gzip`

`bzip`

Examples

general purpose	**gzip**
	bzip
image	**png**
	jpeg

Examples

general purpose	**gzip**
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music	**mp3**

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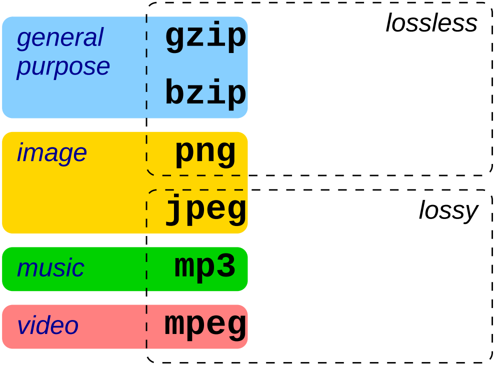
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general purpose
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Examples

		compression ratio	
general purpose	**gzip**	~ 1 : 3	*lossless*
	bzip	~ 1 : 3.5	
image	**png**	~ 1 : 2.5	
	jpeg	~ 1 : 25	*lossy*
music	**mp3**	~ 1 : 12	
video	**mpeg**	~ 1 : 30	

Compression

Is it always possible to compress data?

Theorem

The proportion of binary strings compressible by more than k bits is at most 2^{-k}.

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Proof. For all $n \geq 1$, the number of binary strings of length n is 2^n. The number of binary code strings of length less than $n-k$ is $2^0 + 2^1 + 2^2 + \ldots + 2^{n-k-1} = 2^{n-k} - 1$. Thus the ratio is

$$\frac{2^{n-k}-1}{2^n} < \frac{2^{n-k}}{2^n} = 2^{-k}.$$

$\square$

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Less than 50 % of files are compressible by more than one bit.

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Less than 1 % of files are compressible by more than 7 bits.

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$$\frac{2^{n-k} - 1}{2^n} < \frac{2^{n-k}}{2^n} = 2^{-k}.$$

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Less than 0.000000000000000000000000000001 % of files are compressible by 100 bits.

How is it possible?

Why was the compression ratio greater than one in all the examples we saw?

What are those rare files that are compressible?

Why are the files we use in practice so often compressible?

Compression

`echo <x> | gzip - | wc -c` # multiply by 8 for bits

Source string, x		$\ell(C(x))$	**ratio**
$aaa\ldots a$	$(10000 \times a)$	368	27.2 : 1.

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$aabaabbbbabbbbb\ldots$	(10000 random letters)	13456	$0.74 : 1$

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$abababab\ldots ab$	$(5000 \times ab)$	368	$27.2 : 1$

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$aaa\ldots abbb\ldots b$	$(5000 \times a, 5000 \times b)$	376	$26.6 : 1$

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$abbaababba\ldots$	$(1000 \times abbaababba)$	488	20.5 : 1

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Strings following a rule are compressible?

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$abbaababba\ldots$	$(1000 \times abbaababba)$	488	$20.5 : 1$
$aaabbabbabb\ldots$	$(\pi$, 0–4 $\mapsto a$, 5–9 $\mapsto b)$	13416	$0.74 : 1$

π follows a rule but isn't compressed!

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Maybe it's just `gzip`? It would be possible to create to *special program* to compress π into a short file.

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But what does it mean to compress an *individual* string???

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An individual string is "simple" (as opposed to "complex") if it can be compressed into a small file by a *prespecified* program.

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We can use several compressors if we prefix the code string by an index of the used program.

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Project II

Information vs. Complexity

Is complexity the same as information?

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Is there a lot of *information* in a random string? **No.**

Information vs. Complexity

Is complexity the same as information?

Is there a lot of *information* in a random string? **No.**

$$\begin{aligned} \textit{Complexity} &= \textit{Information} + \textit{Noise} \\ &= \textit{Regularity} + \textit{Randomness} \\ &= \textit{Algorithm} + \textit{Compressed file} \end{aligned}$$

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Reliable communication

What if the code string was transmitter over a *noisy* connection?

Reliable communication

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- Modem line

Reliable communication

What if the code string was transmitter over a *noisy* connection?

- Modem line
- Satellite link

Reliable communication

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- Hard disk

Reliable communication

What if the code string was transmitter over a *noisy* connection?

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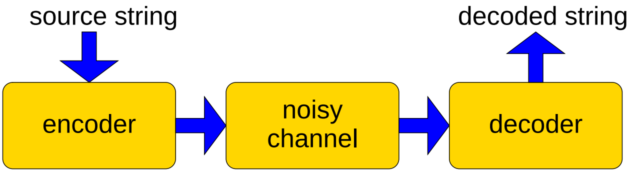
Can we recover the original message (without errors) from a noisy code string?

Error correcting codes



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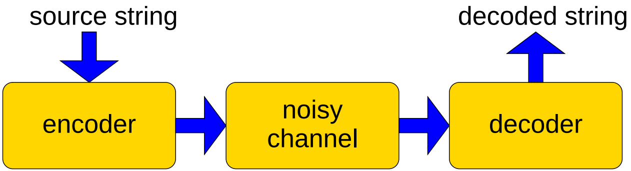

We want to minimize two things:

1. Length of the code string.
2. Probability of error.

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Transmission rate reduced to 1 : 3.

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If errors independent and symmetric, probability of error reduced to $\sim 3p^2$, where p is the error rate of the channel.

Shannon's Theorem

- Binary symmetric channel (BSC), error rate p:

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So what?

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Assume you want to transmit data with probability of error 10^{-15} over a BSC, $p = 0.1$. Using a repetition code, we need to transmit the message **59** times.
(You can check the math yourself. Hint: binomial distribution.)

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