The Sum-Product Bridge

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# Sums of Products United – People

## Club members at the CS department of UH
- Esther Galbrun, *PhD student (co-advisor H. Toivonen)*
- Kustaa Kangas, *MSc student*
- Mikko Koivisto, *leader, advisor*
- Janne Korhonen, *PhD student (co-advisor P. Kaski)*
- Teppo Niinimäki, *PhD student*
- Pekka Parviainen, *PhD student*

## Associate club members (key collaborators)
- Petteri Kaski (*Algodan, Aalto University*)
- Andreas Björklund (*Lund University*)
- Thore Husfeldt (*IT University Copenhagen*)

...
Build and maintain a bridge that connects algorithm theory and computational statistics by developing the methodology of computing large sums of products.
The amazing Fairyland Bridge connects two mountains at 5,000ft at Huangshan.
SoPU – Mission

Build and maintain a bridge that connects algorithm theory and computational statistics by developing the methodology of computing large sums of products.
Probabilistic Models – Sums of Products

Bayesian network

Computational tasks

Inference:
\[ p_G(a|bc) = \frac{\sum_{de} p_G(abcde)}{\sum_{ade} p_G(abcde)} \]

Learning:
\[ G^* \in \text{argmax}_G p_G(abcde)p(G), \]
with \( p(G)=p(G_a)p(G_b)\ldots p(G_e) \)

\[ p_G(abcde)=p(d)p(e)p(a|de)p(b|a)p(c|ae) \]
Sums of Products – Algebra & Combinat.

Algebra

\[ \sum_{x \in A} \prod_S f_S(x_S) \]

- **Rings**
  (+, ⋅) over integers
  (+, ⋅) over polynomials

- **Semirings**
  (max, ⋅)
  (min, +)
  (min, max)

Combinatorics

- The scopes \( S \subseteq \{1, \ldots, n\} \) form a **hypergraph**.
  (E.g., in BN inference)

- The summation is over a domain \( A \subseteq D_1 \times \cdots \times D_n \) that may have a **combinatorial** structure.
  (E.g., in BN learning)
SoPU: Results 2010–2011

- **Algorithm theory**
  - SODA’10,
  - ICALP’10,
  - SODA’12,

- **Computational statistics**
  - AISTATS’10,
  - UAI’11,
  - ECML-PKDD’11,
  - SDM’11.
Interactions: a directed acyclic graph

2012  SODA
2011  UAI  ECML-PKDD  SDM’11
2010  IPL  ICALP  SODA  AISTATS
2009  IWPEC  ESA  UAI
2008  FOCS  STACS  ICALP  WABI
2007  STOC
2006  FOCS  UAI  COLT  IPL  ECML
2005  ICML  JMLR  WABI
2004  Thesis
2003  PSB
Permanent stuff (BHKK, IPL 2010)

- per $A = \sum_p a_{1p(1)} \cdots a_{mp(m)}$, where $p$ runs through all injections from $[m]$ to $[n]$.

**Theorem**

<table>
<thead>
<tr>
<th>Algebraic structure</th>
<th>Time complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>semiring</td>
<td>$m \ B(n, m)$</td>
</tr>
<tr>
<td>commutative semiring</td>
<td>$m(n-m+1)2^m$</td>
</tr>
<tr>
<td>ring</td>
<td>$m \ B(n, m/2)$</td>
</tr>
<tr>
<td>commutative ring</td>
<td>$(mn-m^2+n)2^m$</td>
</tr>
</tbody>
</table>

$B(n, m)$ is the number of subsets of $[n]$ of size at most $m$. 
What Next

- Keep the main themes
  - Make use of subtraction (additive inverses)
  - Optimization via counting
  - Space-time tradeoff considerations
- Bilinear transforms
  - Systematic study
- Bayesian networks
  - Implement into a public software
  - Apply to causal discovery with domain experts
- Other
  - Can randomized algorithms be much faster?
  - Better combinatorial bounds?