Below are a wavelet tree for the BWT of ABRACADABRA$ and the function $C(a)$ that returns the partial sums of the characters’ frequencies. Suppose each node $v$ supports the queries $v.\text{rank}_0(i)$ and $v.\text{rank}_1(i)$ on the binary sequence it stores. Considering the tree as an FM-index, list the rank queries used to count the number of occurrences of BRA in ABRACADABRA$.

\[
\begin{array}{c|cccccc}
 a & $ & A & B & C & D & R \\
 C(a) & 0 & 1 & 6 & 8 & 9 & 10 \\
\end{array}
\]

\[\text{SOLUTION:}\]

We can see from $C$ that there are 5 As in $\text{BWT}(S)$, so the ranks of the first and last A in $\text{BWT}(S)[1..12]$ are 1 and 5. Adding $C[A] = 1$ to 1 and 5 we get 2 and 6, meaning the lexicographically 2nd through 6th suffixes of $S$ start with A.\(^1\)

To compute the rank $\text{BWT}(S).\text{rank}_R(2 - 1) + 1 = 1$ of the first R in $\text{BWT}(S)[2..6]$, we compute

- $v_1.\text{rank}_1(1) = 0$,
- $v_3.\text{rank}_1(0) = 0$.

To compute the rank $\text{BWT}(S).\text{rank}_R(6) = 2$ of the last R in $\text{BWT}(S)[2..6]$, we compute

- $v_1.\text{rank}_1(6) = 3$,
- $v_4.\text{rank}_1(3) = 2$.

Adding $C[R] = 10$ to 1 and 2 we get 11 and 12, meaning the lexicographically 11th through 12th suffixes of $S$ start with RA.

To compute the rank $\text{BWT}(S).\text{rank}_R(11 - 1) + 1 = 1$ of the first B in $\text{BWT}(S)[11..12]$, we compute

- $v_1.\text{rank}_1(10) = 7$,
- $v_2.\text{rank}_1(7) = 1$.

\(^1\)You can get the same result by computing the ranks for first and last A in $\text{BWT}(S)[1..12]$.
- \( v_5.rank_0(1) = 0. \)

To compute the rank \( BWT(S).rank_B(12) = 2 \) of the last B in \( BWT(S)[11..12] \), we compute

- \( v_1.rank_1(12) = 9, \)
- \( v_2.rank_1(9) = 3, \)
- \( v_5.rank_0(3) = 2. \)

Adding \( C[B] = 6 \) to 1 and 2 we get 7 and 8, meaning the lexicographically 7th and 8th suffixes of \( S \) start with BRA.

Since 8 - 7 + 1 = 2, we report that there are 2 occurrences of BRA.