A Constraint Optimization Approach to Causal Discovery from Subsampled Time Series Data

Antti Hyttinen
Joint work with Sergey Plis, Matti Järvisalo, Frederick Eberhardt, David Danks

University of Helsinki, Helsinki Institute for Information Technology
Mind Research Network and University of New Mexico
California Institute of Technology
Carnegie Mellon University

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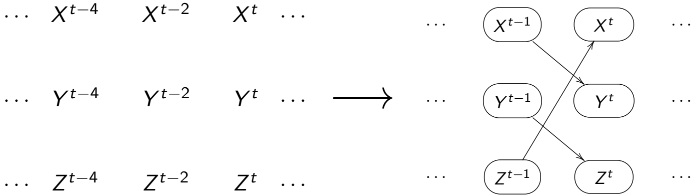
Problem Statement

How to discover the causal structure at the **system timescale** from time series data obtained at a coarser **measurement timescale**?



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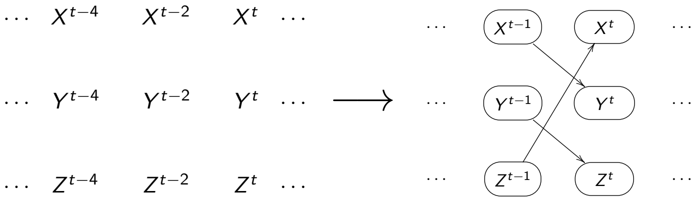

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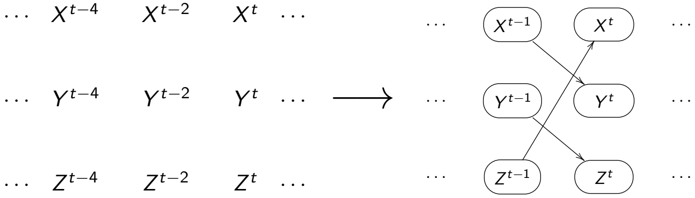

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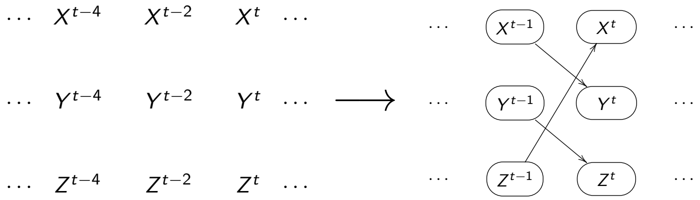

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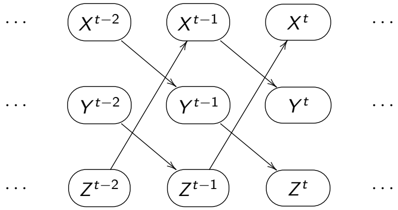
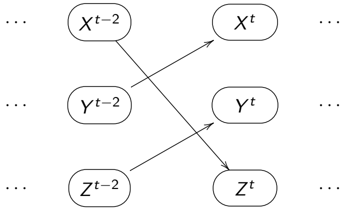
- Subsamping induces confounding, and unidentifiability
- Applications: e.g. fMRI.

Subsampling needs to be taken into account!



True structure at the system timescale



measurement time scale structure

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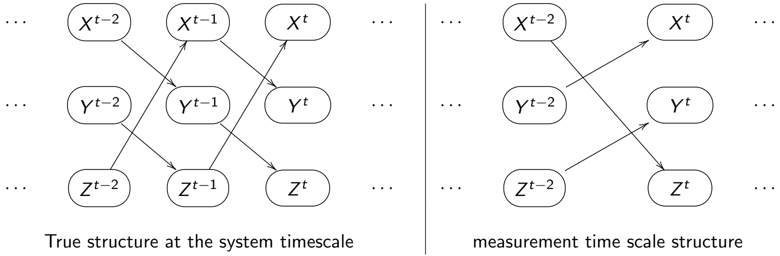

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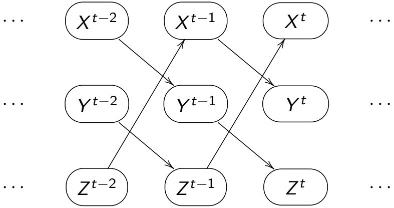

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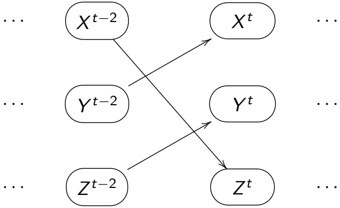

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When ignoring subsampling:

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- Wrong result for interventions.

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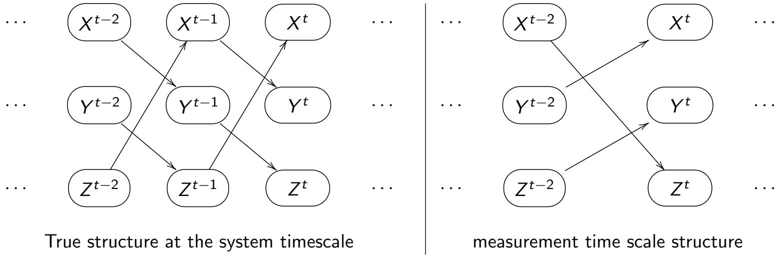


When ignoring subsampling:

- All direct causal relationships misspecified.
- Wrong result for interventions.
- Wrong interventions suggested.

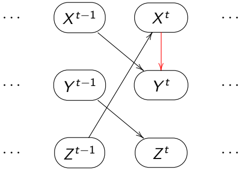
Overview

1. Previous Literature
2. Graphical Representation
3. A Constraint Satisfaction Solution
4. A Constraint Optimization Solution
5. Conclusion

Previous Literature

Previous Literature 1

- Adding instantaneous effects in a linear model (see for example Lütkepohl 2005 or Hyvärinen et al 2010).



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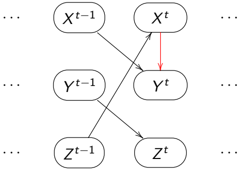

- Continuous time approaches, but some processes are inherently discrete time (e.g. salary payment).

Previous Literature 2

Recently Plis et al. (UAI2015,NIPS2015) considered modeling subsampling directly, assuming on the system timescale level:

- discrete time
- first order Markov: $\mathbf{V}^t \perp\!\!\!\perp \mathbf{V}^{t-k} | \mathbf{V}^{t-1}$
- no instantaneous effects, or unobserved common causes
- nonparametric (continuous or discrete values, SVAR processes, or dynamic BNs)
- Measurements from this at integer intervals (e.g. every second).

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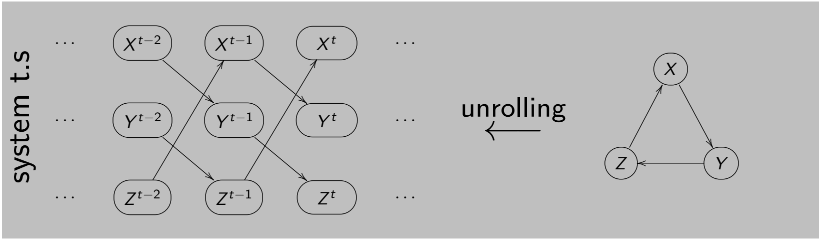
Corresponding parametric method: Gong et al. (ICML2015) discovered linear models using non-Gaussianity.

Graphical Representation

Rolled Representation



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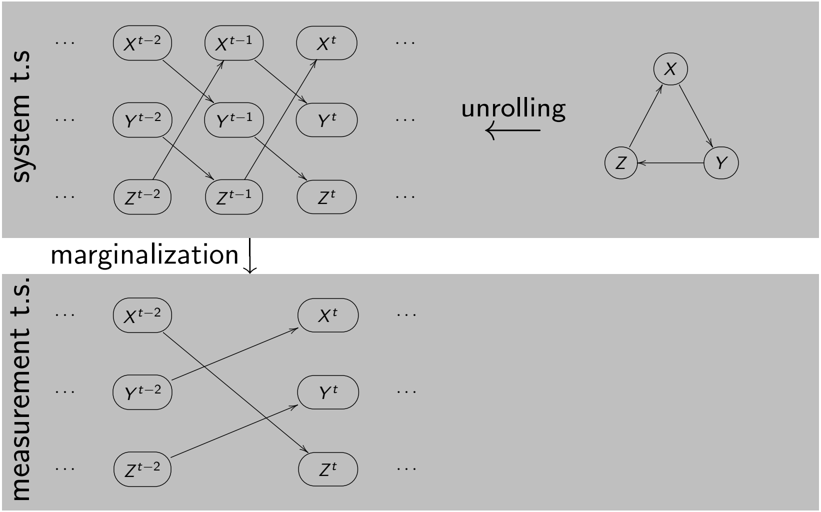

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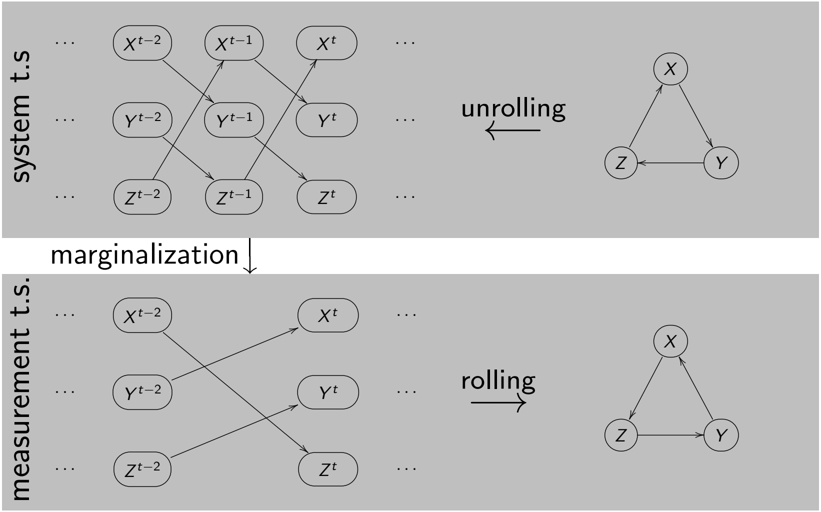
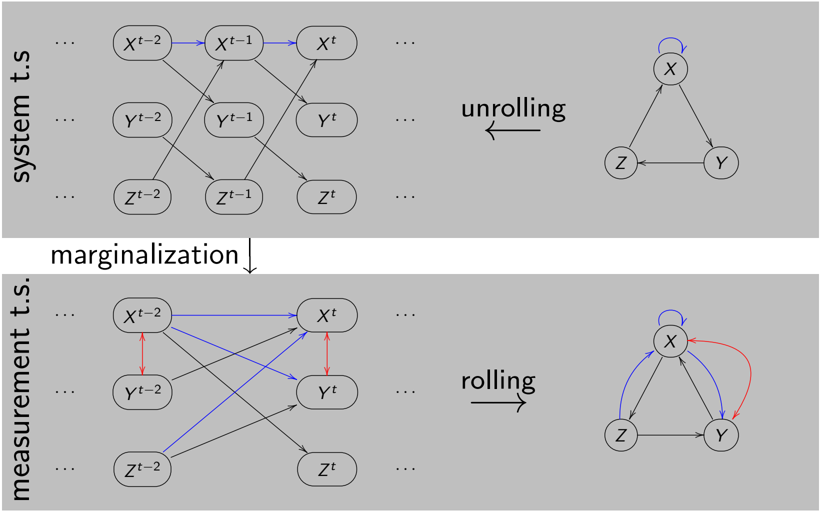


Induced confounding



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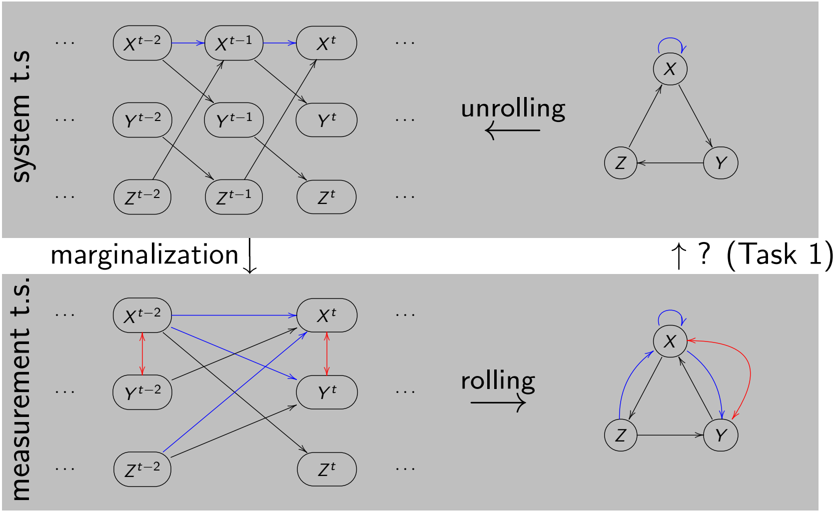
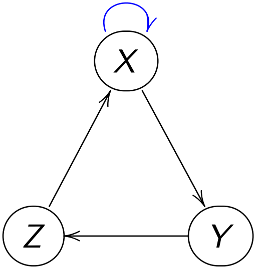
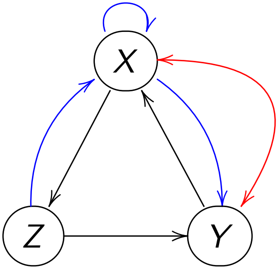


Correspondence between System and Measurement T.S.



system timescale



measurement timescale

When subsampling by u:

- Measurement time scale edge $Y \to X$ corresponds to path of length u: $Y \to \cdots \to X$
- Measurement time scale edge $X \leftrightarrow Y$ corresponds to paths of length $k < u$: $W \to \cdots \to X$ and $W \to \cdots \to Y$.

A Constraint Satisfaction Solution

Partial Complexity Result

Result: Deciding whether there is a system t.s. structure compatible with the directed edges of a measurement t.s. structure is **NP-complete** for any fixed $u \geq 2$.

Proof: Binary matrix root.

A Constraint Satisfaction Solution

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- The symbolic encoding gets grounded.
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- Backtracking DFS by `Clingo` (Gebser et al. 2011).

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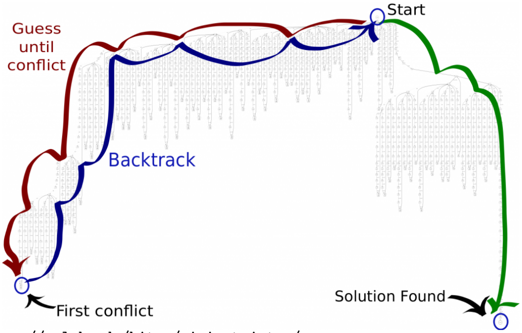
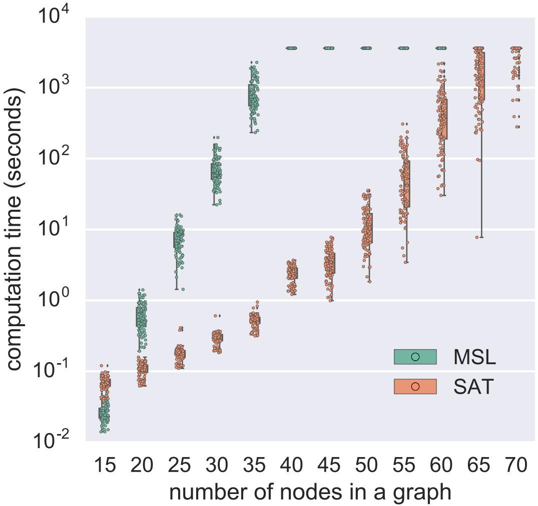
- You write a symbolic encoding.
- The symbolic encoding gets grounded.
- The encoding gets turned into conjunctive normal form.
- Backtracking DFS by `Clingo` (Gebser et al. 2011).
- Exact and complete solution.
- Subsampling rate u: fixed or free.



https://srlabs.de/bites/minisat-intro/

```
node(1..3). % Measurement timescale structure
edgeh(1,2).no_edgeh(1,3).confh(2,3).no_confh(1,2). %and so on

urange(1..5).  % Define a range of u:s
1 { u(U): urange(U) } 1. % u(U) is true for only one U

{ edge1(X,Y) } :- node(X), node(Y). %draw G1

% Derive all directed paths up to length U
path(X,Y,1) :- edge1(X,Y).
path(X,Y,L) :- path(X,Z,L-1), edge1(Z,Y), L <= U, u(U).

% Determine measurement t.s. for G1
edgeu(X,Y) :- path(X,Y,L), u(L).
confu(X,Y) :- path(Z,X,L), path(Z,Y,L), node(X;Y;Z),
                               X < Y, L < U, u(U).
% Check consistency
:- edgeh(X,Y), not edgeu(X,Y). :- no_edgeh(X,Y), edgeu(X,Y).
:- confh(X,Y), not confu(X,Y).  :- no_confh(X,Y), confu(X,Y).
```

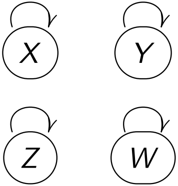
Scalability of Enumerating 1000 Solutions



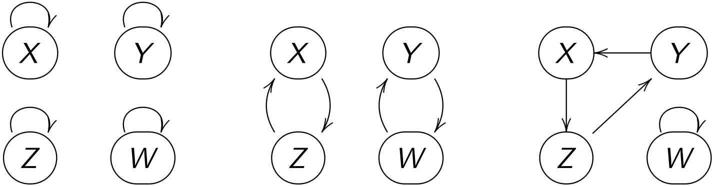
(fixed subsampling rate 2, SAT is our approach,
MSL is the previous state of art by Plis et al. (2015))

Identifiability: Underdetermination

Measurement timescale structure:



could be produced by system timescale structures:

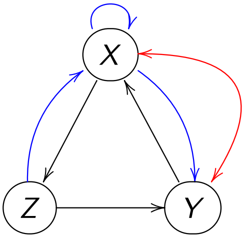


$u = 1, 2, 3, \cdots$ $\quad$ $u = 2, 4, 6, \cdots$ $\quad$ $u = 3, 6, 9, \cdots$

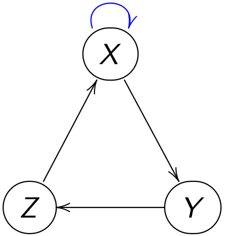
or a four cycle in either direction and symmetrically!

Identifiability: An Identified Case

But measurement timescale structure:



uniquely identifies system timescale structure



and the subsampling rate $u = 2$.

A Constraint Optimization Solution

Task 2: Finding Structures Compatible with Data

$$
\begin{array}{ccccc}
\cdots & X^{t-4} & X^{t-2} & X^{t} & \cdots \\
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\longrightarrow
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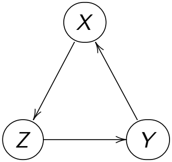

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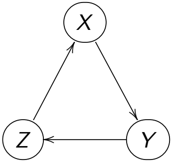

data

measurement t.s.

system t.s.

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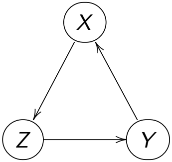

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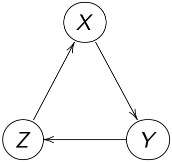

data — measurement t.s. — system t.s.

- Measurement t.s. structure can be consistently estimated from data under faithfulness: e.g.

$$X \to Z \quad \Leftrightarrow \quad X^{t-u} \not\perp\!\!\!\perp Z^{t} \mid \mathbf{V}^{t-u} \setminus X^{t-u}$$
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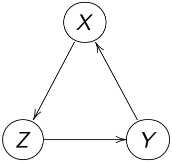

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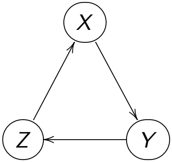

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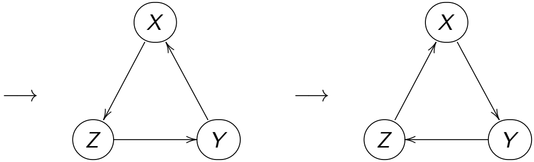

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- Due to finite samplesize, the constraint satisfaction approach will often return UNSATISFIABLE.
- Find the system t.s. structure such that its measurement t.s. structure is optimally close to the estimated (Task 2).

A Constraint Optimization Solution

Specifics:

- Penalize inconsistencies between absences and precences of edges in the measurement t.s.:
  - Either uniform weights, or
  - log Bayesian probabilities of the corresponding (in)dependence, obtained through Bayesian model selection (see Hyttinen et al. 2014)
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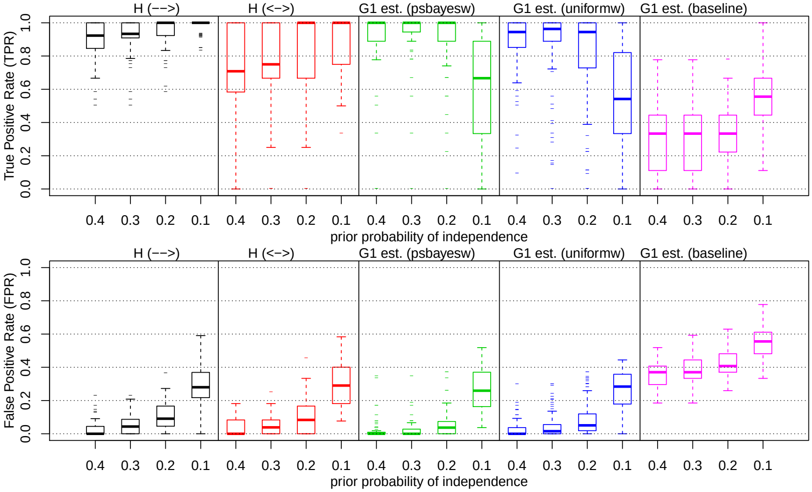
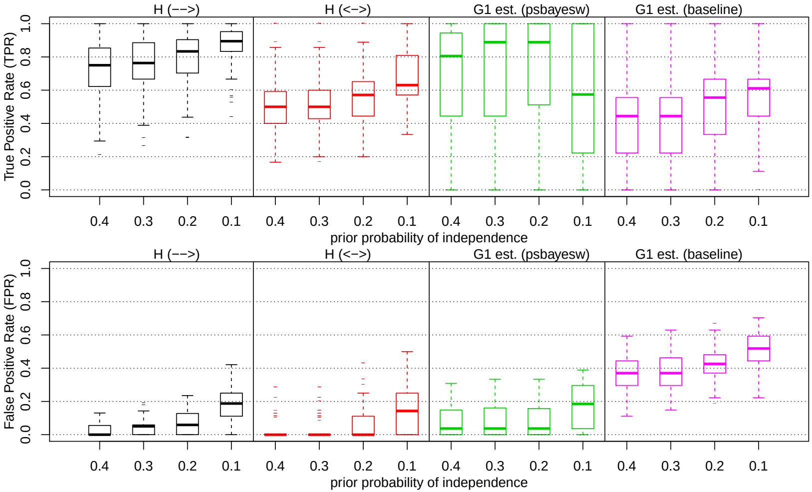
Accuracy for fixed $u = 2$



(fixed subsampling rate 2, average result of the eq. class,
6 nodes, av. degree 3, 200 samples, 100 data sets, linear models)

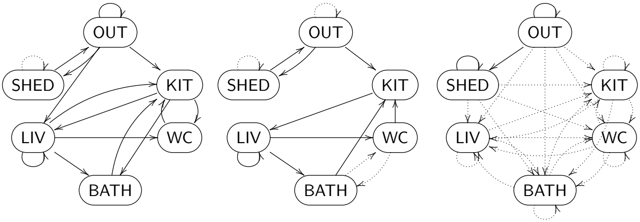
Accuracy for $u = 3$



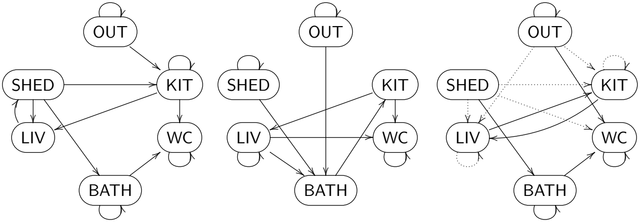
Analysis of Temperature/Humidity data 1

- Hourly measurements of six sensors placed in a house.
- Temperature and humidity recorded.
- Removed trends.
- Handle undetermination: for each edge [Magliacane et al.]
  - run the inference procedure enforcing presence
  - and then enforcing absence
  - difference in objectives gives the support for the edge.

Analysis of Temperature/Humidity data 2



a) Temperature at $u = 2$ b) Temperature at $u = 3$ c) Temperature at $u = 10..12$



d) Humidity at $u = 2$ e) Humidity at $u = 3$ f) Humidity at $u = 10..12$

Edges with full lines are found to be present, absent edges are found to be absent, edges with dotted lines are present or absent.

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Thanks!