Causal Discovery from Subsampled Time Series Data by Constraint Optimization

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6.9.2016

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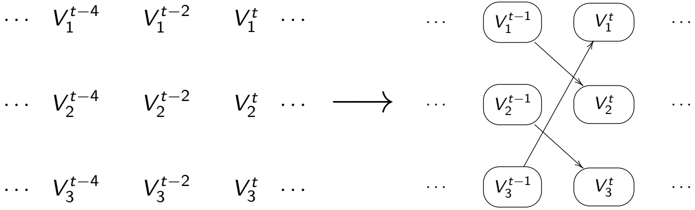

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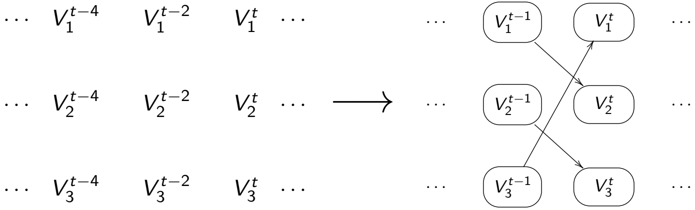

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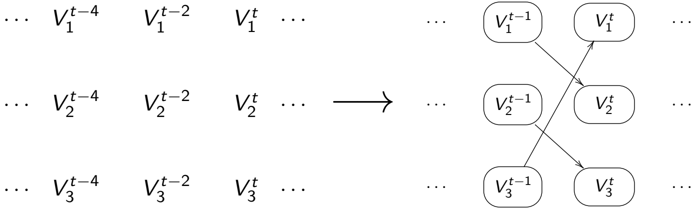

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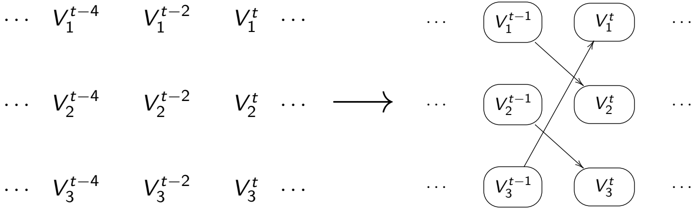

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Introduction to Subsampling

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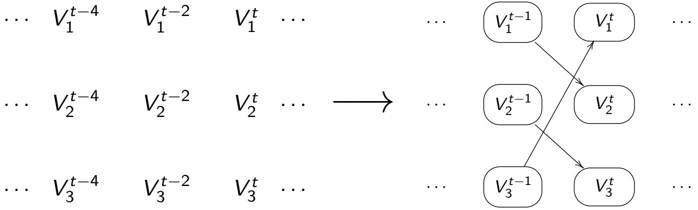

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- Applications: e.g. fMRI.

Previous Literature

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  - discrete time
  - first order Markov: $\mathbf{V}^t \perp\!\!\!\perp \mathbf{V}^{t-k} | \mathbf{V}^{t-1}$
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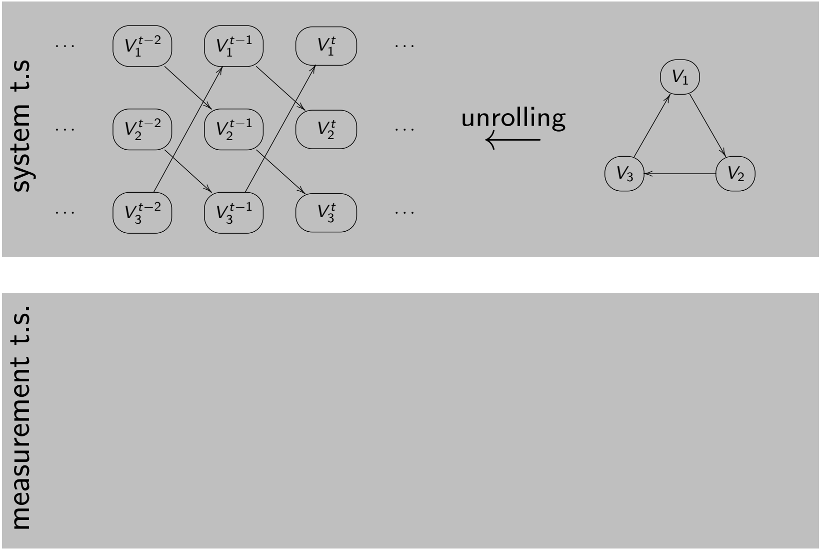
- Corresponding parametric method: Gong et al. (ICML2015) discovered linear models using non-Gaussianity.

Rolled Representation



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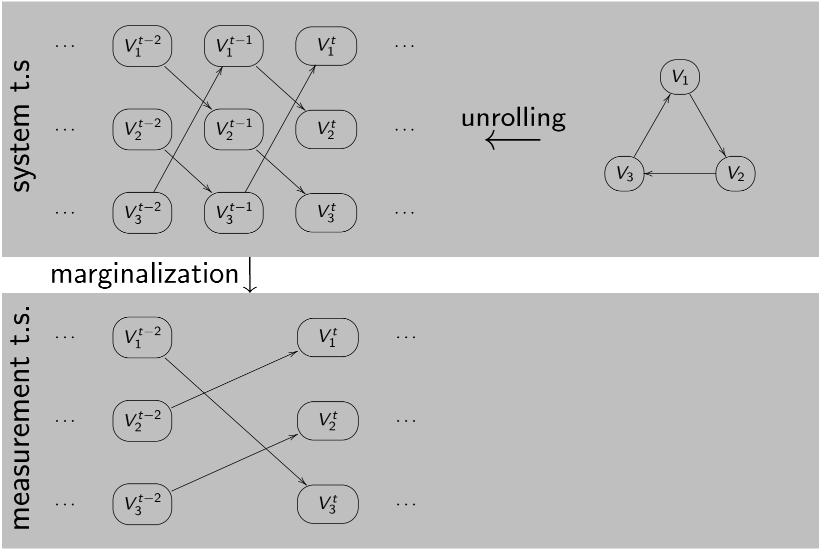

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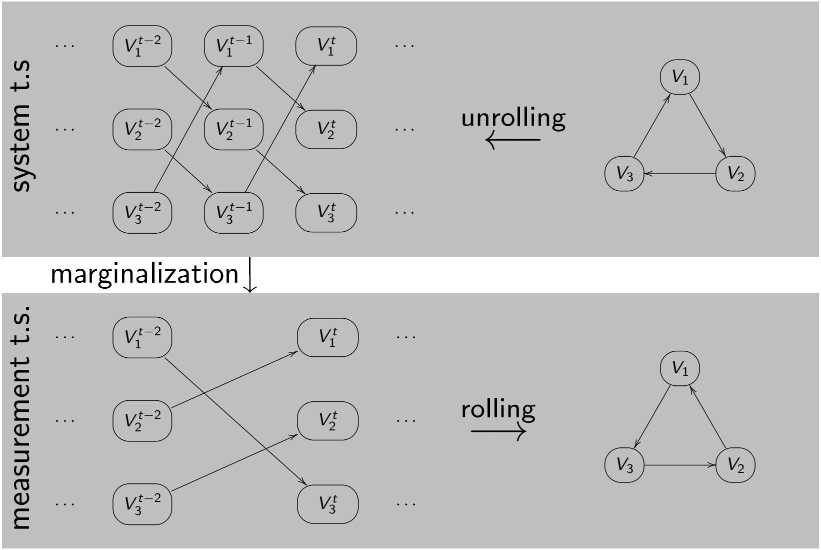

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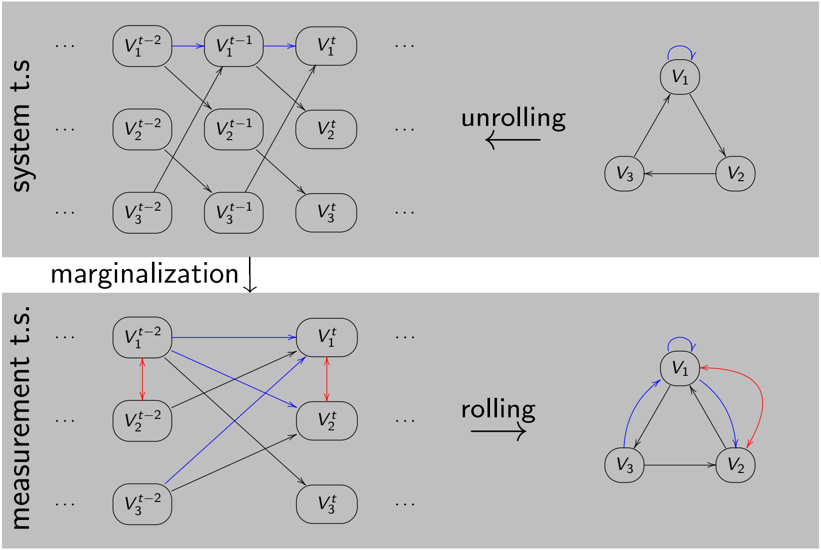

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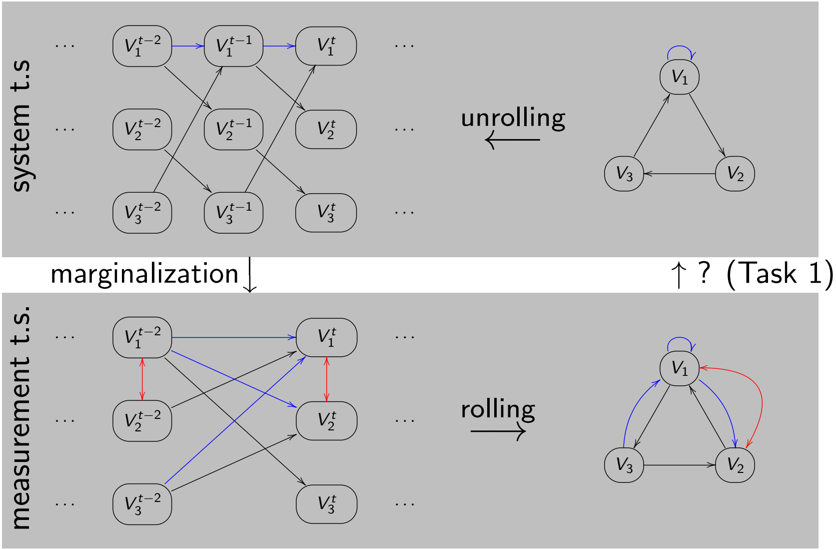

Task 1: Finding System Timescale Structures

Result 1: Deciding whether there is a system t.s. structure compatible with the directed edges of a measurement t.s. structure is **NP-complete** for any fixed $u \geq 2$.

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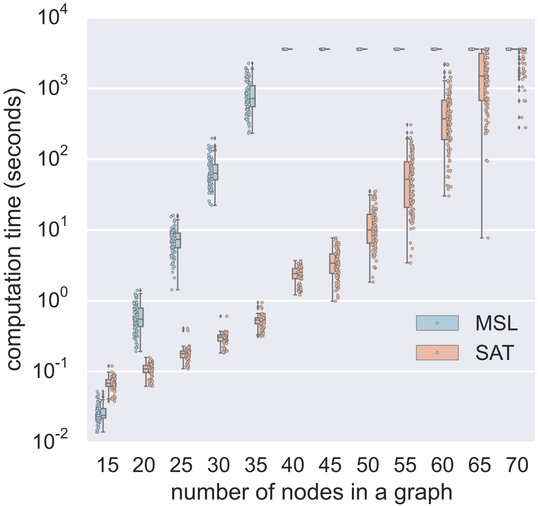
Result 2: A constraint satisfaction solution by ASP:

- We encoded the problem (the marginalization operation) using the expressive declarative modeling language
- Solver `Clingo` (Gebser et al. 2011) uses state-of-the-art SAT-solving techniques to give an exact and complete solution
- ASP is relatively easy and quick to use, the encoding is easily extendable
- Subsampling rate u: fixed or free.

Scalability of Enumerating 1000 Solutions



(fixed subsampling rate 2, SAT is our approach,
MSL is the previous state of art by Plis et al. (2015))

Task 2: Finding Structures Compatible with Data

$$\begin{array}{cccccc}
\cdots & V_1^{t-4} & V_1^{t-2} & V_1^{t} & \cdots \\
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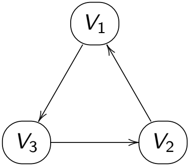

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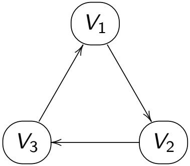

data

measurement t.s.

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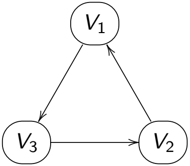

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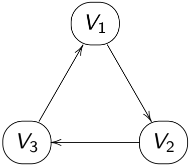

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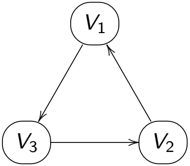

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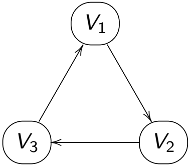

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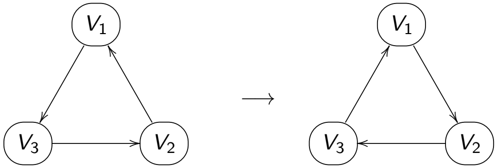

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- Due to finite samplesize, the constraint satisfaction approach will often return UNSATISFIABLE
- Find the system t.s. structure such that the corresponding measurement t.s. structure is optimally close to the estimated (Task 2).

Result 3: A Constraint Optimization Solution

Specifics:

- Penalize inconsistencies between absences and precences of edges in the measurement t.s.:
  - Either uniform weights, or
  - log Bayesian probabilities of the corresponding (in)dependence, obtained through Bayesian model selection (see Hyttinen et al. 2014)
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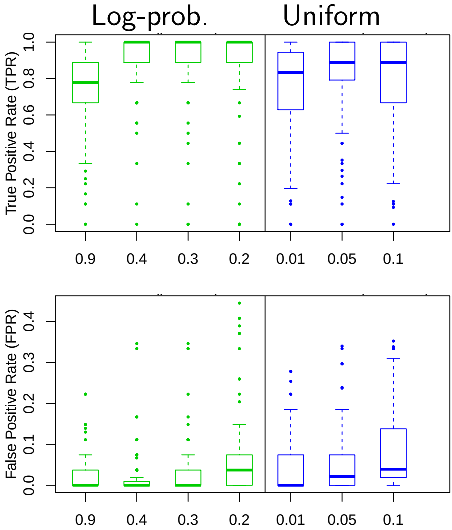
- Previous work by Plis et al. 2015: searching neighbors of the estimated measurement t.s. structure — resembles the uniform weighting scheme.

Accuracy of the Estimated System Timescale Edges



(fixed subsampling rate 2, average result of the eq. class,
6 nodes, av. degree 3, 200 samples, 100 data sets, linear models)

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Thanks!