Model Answers – Chapters 6 and 7

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Exercise 6.4

(a) Player B has the strictly dominant strategy of playing $L$. This strategy will result in the highest pay-off for player B regardless of what player A does. Player A has no dominant strategy.

(b) The only pure strategy Nash equilibrium is $(b, L)$.

Exercise 6.7

(a) $(D, R)$ is the only Nash equilibrium. This game has no mixed strategy Nash equilibria.

(b) This game has no pure strategy Nash equilibria, but the following mixed strategy Nash equilibrium exists:

\[
5q + 0(1 - q) = 4q + 2(1 - q) \Rightarrow q = \frac{2}{3}
\]

\[
6p + 4(1 - p) = 10p + 2(1 - p) \Rightarrow p = \frac{1}{3}
\]

Exercise 6.15

\[
\begin{array}{c|ccc}
 & A & B & \text{no} \\
\hline
A & -10, -10 & 10, 10 & 15, 0 \\
B & 10, 10 & 5, 5 & 30, 0 \\
\text{no} & 0, 15 & 0, 30 & 0, 0 \\
\end{array}
\]

(b) True. The worst case outcome is making a profit of 5 million when producing $B$.

(c) False. If firm 2 entered producing $B$, the best response for firm 1 would still be to produce $A$ instead. Both firms entering the market producing $B$ does not make it a Nash equilibrium.

(d) Firm 1 producing $A$, and firm 2 producing $B$, or firm 1 producing $B$ and firm 2 producing $A$.

(e) The firms would reach social optimality if they by merging could produce only $B$ (and if necessary limit the production output, etc.). This would result in a total profit of 30 million. This compares favourably to the 5 million each would be guaranteed to make by entering the market and producing $B$, or the 10 million each could make in one of the pure strategy Nash equilibria.
There is a possibility that the other firm would stay out of the market without a merger, but this seems highly unlikely given that both firms are guaranteed a minimum profit of 5 million by entering and producing $B$.

Exercise 7.4

(a) $(X, X)$ and $(Y, Y)$ are Nash equilibria for $x \in \{0, 1\}$. For $x = 2$, $(Y, Y)$ is the only Nash equilibrium. $Y$ is an evolutionarily stable strategy for $x \in \{0, 1, 2\}$, and additionally $X$ is an evolutionarily stable strategy for $x = 0$.

(b) According to the definition in section 7.3, a strategy in such a game is evolutionarily stable if and only if (i) $a > c$ or (ii) $a = c$ and $b > d$. If the strategy $X$ is weakly dominated, the strategy $Y$ has to be dominant because the game has only two strategies. According to the given definition of a weakly dominated strategy, it then has to be the case that $c = a$ (if $c > a$, $(X, X)$ would not be a Nash equilibrium, and if $c < a$, $Y$ would not be a dominant strategy). Thus, (i) cannot be true.

If we assume that (ii) holds, it must be that $b > d$. But if $b > d$, $Y$ is no longer a dominant strategy, because the pay-off for $(X, Y)$ would be higher than the pay-off for $(Y, Y)$, and thus $X$ is not weakly dominated. It follows that neither (i) nor (ii) can hold when $X$ is weakly dominated, and thus $X$ cannot be evolutionarily stable when $X$ is weakly dominated.