58131 Data Structures (Spring 2009)

Homework 2 (19.–23.1.2009)

At the second homework session, small groups will be formed for the group works. In the group works, it is important that the whole group can meet. Please attend your "own" homework group this time. If not, please contact your group teacher. In forming the groups, it is a good idea to select members on the basis of common available meeting times.

1. (a) The following algorithm evaluates the polynomial function \( p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_k x^k \), when the constant coefficients \( a_0, \ldots, a_k \) are stored in an array \( A[0..k] \):

   ```
   EVALUATE-POLYNOMIAL(A[0..k], x)
   1   \( s \leftarrow 0 \)
   2   \textbf{for} \( i \leftarrow 0 \) \textbf{to} \( k \) \textbf{do}
   3       \( z \leftarrow 1 \)
   4       \textbf{for} \( j \leftarrow 1 \) \textbf{to} \( i \) \textbf{do}
   5           \( z \leftarrow z \cdot x \)
   6       \( s \leftarrow s + A[i] \cdot z \)
   7   \textbf{return} \( s \)
   ```

   What is the running time of the algorithm? (Assume arithmetic operations take a constant time.) What is the loop invariant for the outer \textbf{for} loop?

   (b) The algorithm given in the previous problem for evaluating a polynomial function could be made more efficient. The outline for a more efficient algorithm would be

   ```
   EVALUATE-POLYNOMIAL-2(A[0..k], x)
   \ldots
   \textbf{for} \( i \leftarrow 0 \) \textbf{to} \( k \) \textbf{do}
   \( z = x^i \text{ and } s = \sum_{j=0}^{i-1} A[j]x^j \)
   \ldots
   \textbf{return} \( s \)
   ```

   where a new loop invariant has been written in as a comment. Fill in the missing parts of the algorithm so that the loop will indeed have the given invariant.

2. We know that a certain algorithm takes 1 ms to run for input size \( n = 100 \). How long does it take for input size 10000, when the running time of the algorithm is

   \( (a) \ \Theta(\log n) \)
   \( (b) \ \Theta(n \log n) \)
   \( (c) \ \Theta(n^2) \)
   \( (d) \ \Theta(2^n) \)?

   In each case assume that the lower order terms can be neglected. For example, in part (c) assume that the running time is \( cn^2 \) for some constant \( c \).

   \textbf{Continues on the next page!}
3. Which of the following claims are true, which not? Give a motivation for your answer.

(a) \( \log n = O(10^6) \)
(b) \( 10^6 = O(1) \)
(c) \( \log(n^2) = \Theta(\log n) \)
(d) \( \log(n^2) = \Omega((\log n)^2) \)
(e) \( 3^n = O(2^n) \)
(f) \( \sum_{i=0}^{n} i^k = \Theta(n^{k+1}) \)

4. Prove that \( f = \Theta(g) \) if and only if \( f = O(g) \) and \( g = O(f) \).

5. Let \( f, g : \mathbb{N} \to \mathbb{R}^+ \cup \{0\} \). Define \( f(n) = o(g(n)) \), if \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \). Prove that if \( f = o(g) \), then \( f = O(g) \), but \( g \neq O(f) \).