Finding frequent (closed) sets with tree structures

FP-tree, FP-growth, CLOSET

- FP-tree data structure
- FP-growth algorithm for finding all frequent sets
- CLOSET algorithm for finding frequent closed sets
- Literature for this part

Key properties

Problem: discovery of frequent sets

- a compressed representation of the database (FP-tree)
- no explicit generation of candidates
- recursive partitioning of search space

Key ideas

- scan database once, compute the frequenies of singletons
- scan the database for a second time and store it as a tree, also store counts in the tree
- while building the tree, prune and sort items by their frequency (try to minimize the tree size)
- determine frequent sets using the tree, without accessing the database again

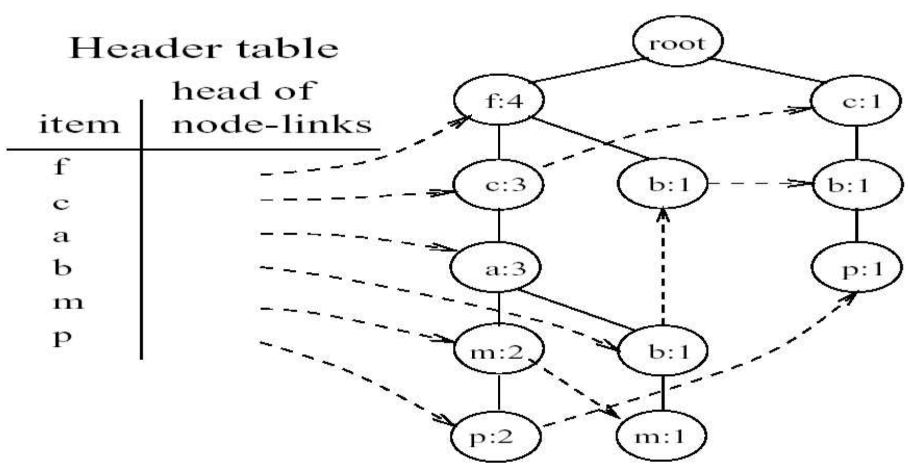
Example relation

(here $a, b, \ldots \in R$ are items)

Row	Ordered frequent items
a, c, d, f, g, i, m, p	f, c, a, m, p
a, b, c, f, l, m, o	f, c, a, b, m
b, f, h, j, o	f, b
b, c, k, p, s	c, b, p
a, c, e, f, l, m, n, p	f, c, a, m, p

Frequency threshold = 3/5.

FP-tree



Constructing FP-tree

- first database scan: frequent sets and absolute frequencies are $f : 4$, $c : 4$, $a : 3$, $b : 3$, $m : 3$, $p : 3$
- initialize the FP-tree (frequent pattern tree) T:
  $T =$ node labeled "null"
- second database scan: for each row
  - read the row
  - remove infrequent items and sort the frequent ones in descending order by frequency
  - add the resulting string to T, update counts as necessary

FP-tree data structures

- a tree, with the root labeled "null", and with paths in the tree representing item prefixes
- links across the tree, linking all occurrences of the same item in the tree
- each node (except null) consists of
  - item name: item identifier
  - count: nr of rows reaching this node
  - node link: link to next node in the tree with the same item identifier
- frequent item header table: starting point for the cross links

String insertion procedure

- **procedure insert_tree(string $[p|P]$, tree rooted at T)**
- p is the first item of the string and P is the remaining string
- the 2nd database scan: for each row $t \in r$ call insert_tree(t', T), where t' is the pruned and resorted contents of the row, and T is the root of the tree

```
1. if T has a child node N such that N.itemname = p then
2.      N.count++;
3. else
4.      create a new node N;
5.      N.itemname := p; N.count := 1;
6.      update nodelinks for p to include N;
7. if P is non-empty
8.      call insert_tree(P, N);
```


Analysis

- Time complexity:
  - 2 scans over the database
  - tree building: $\mathcal{O}(||r||)$ (total number of items)
- Space complexity:
  - $\mathcal{O}(||r||)$
  - average complexity much better!? (pruning and sorting of items)
  - tree height bounded by the size of the maximal row

Finding frequent sets

FP-growth algorithm

- for all frequent items A, in increasing order of frequency (i.e., starting from the bottom of the header table and the tree):
  - traverse all occurrences of A in the tree using the node links
  - at each node N with N.itemname $= A$, determine the frequent sets in which A occurs
  - do this by only looking at the path from root to N (all sets including nodes below N have been generated already in earlier iterations)

Example

item p

- two paths
  - $f:4$, $c:3$, $a:3$, $m:2$, $p:2$
  - $c:1$, $b:1$, $p:1$
- i.e., $fcam$ occurs twice with p and cb once; p's frequency is 2+1=3
- $\Rightarrow p$'s *conditional pattern base* (note: p removed, counts adjusted):
  - $f:2$, $c:2$, $a:2$, $m:2$
  - $c:1$, $b:1$
- frequent sets that contain p are determined by the conditional pattern base!

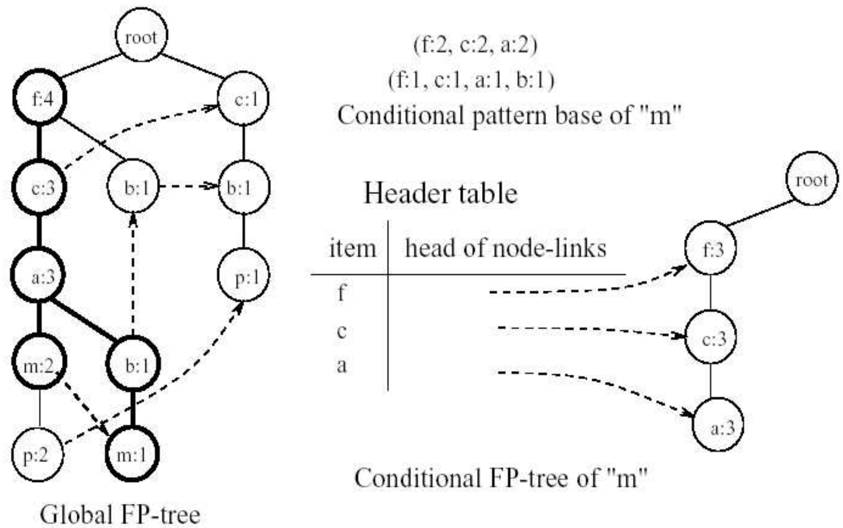
- $\Rightarrow$ recursively build an FP-tree for the conditional base and find frequent sets there, then add p to them all
- the recursive call:
- conditional "data base" given as input
  - $f:2$, $c:2$, $a:2$, $m:2$
  - $c:1$, $b:1$
- 1st database scan: only $c:3$ is frequent
- $\Rightarrow$ cp is frequent, frequency 3/5

Example

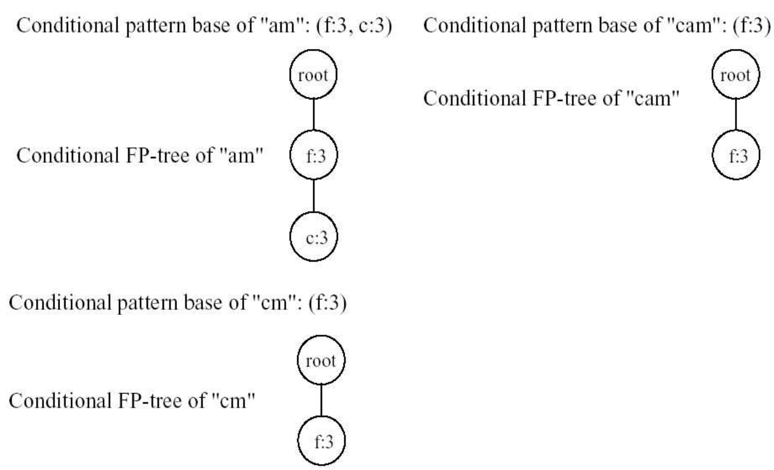
item m

- two paths
  - $f:4$, $c:3$, $a:3$, $m:2$
  - $f:4$, $c:3$, $a:3$, $b:1$, $m:1$
- p can now be ignored, sets containing it were found already
- m's conditional pattern base:
  - $f:2$, $c:2$, $a:2$
  - $f:1$, $c:1$, $a:1$, $b:1$
- m's conditional FP-tree: just one path $f:3$, $c:3$, $a:3$
- recursively find frequent patterns in the conditional FP-tree, first for a, then for c and f

Example, conditional FP-tree of m



Example, conditional FP-trees of ma and mac



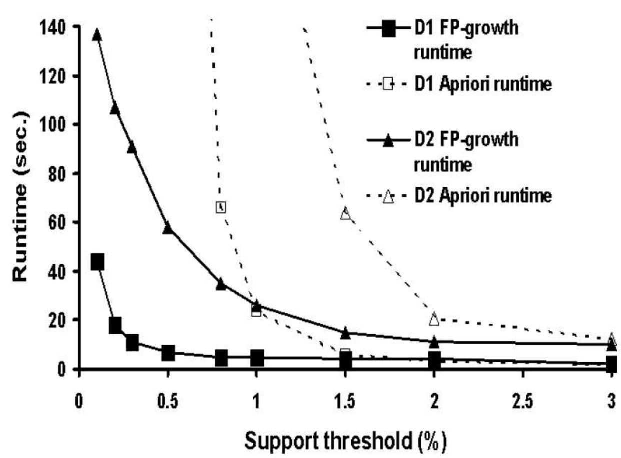
Observation: if the tree consists of just one path, one can simply generate all combinations of items

FP-growth algorithm

Algorithm FP-growth((conditional) FP-tree T, condition $X \subseteq R$)
First call: FP-growth(root, $\emptyset$)

1. **if** tree T consists of a single path **then**
2.   **for** all combinations Y of items in the path
3.   output set $X \cup Y$ and the minimum count of nodes in Y;
4. **else for** each item A in the header table of T
5.   output $Z := X \cup \{A\}$ and the count A.count;
6.   construct Z's conditional FP-tree T_Z;
7.   **if** $T_Z \neq \emptyset$ **then** call FP-growth(T_Z, Z);

Experimental results



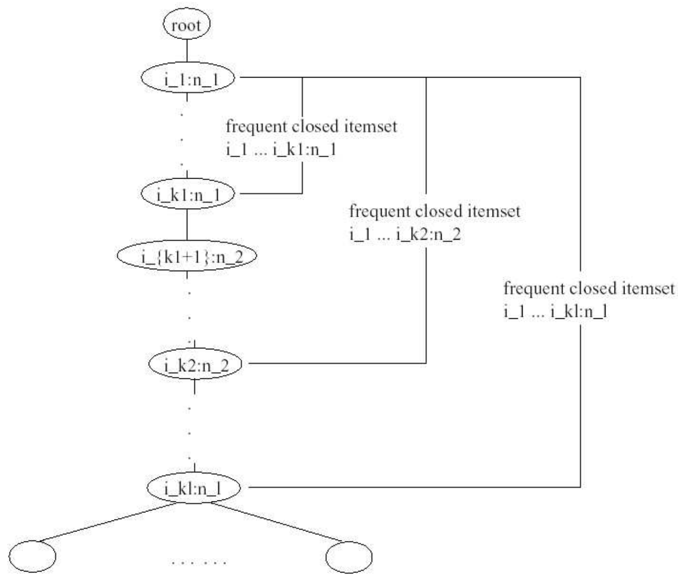
CLOSET: closed sets with FP-tree

- use conditional pattern bases to locate closed sets
- **Lemma** Consider the conditional database of some set X and the (possibly empty) set Y of items that appear in every row of the conditional database. $X \cup Y$ is closed if no closed set Z has been yet found such that $X \cup Y \subset Z$ and Z's count is identical to Y's count in the conditional database.

CLOSET optimizations to FP-growth

- the set Y of items that appear in every row of the conditional database — if not empty — is a prefix of the only path from the root of the conditional FP-tree
  $\Rightarrow$ handle these directly, not recursively
- in more general, if there exists a single prefix path from the root, possibly several closed sets can be extracted directly
- if $X \subset Y$, the counts are equal, and Y is closed, then there are no closed sets that contain X but not Y
  $\Rightarrow$ such sets X can be pruned

Single path optimization



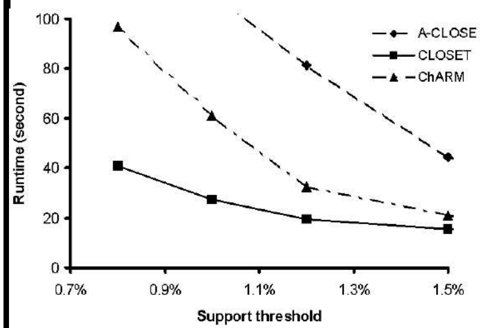
Experimental results

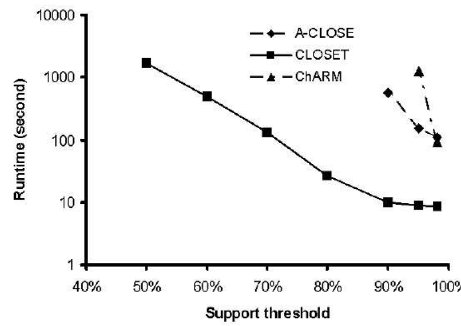
- Again: number of frequent closed set vs. frequent sets

Support	#F.C.I	#F.I	$\frac{\#F.I}{\#F.C.I}$
64179 (95%)	812	2,205	2.72
60801 (90%)	3,486	27,127	7.78
54046 (80%)	15,107	533,975	35.35
47290 (70%)	35,875	4,129,839	115.12

Table 2: The number of frequent closed itemsets and frequent itemsets in dataset *Connect-4*.(F.C.I for *frequent closed itemsets* and F.I for *frequent itemsets.*)

- Performance comparision with other algorithms for closed frequent sets





Literature

- FP-tree, FP-growth:
  Jiawei Han, Jian Pei, Yiwen Yin: Mining Frequent Patterns without Candidate Generation. 2000 ACM SIGMOD Intl. Conference on Management of Data.
- CLOSET:
  Jian Pei, Jiawei Han, Runying Mao: CLOSET: An Efficient Algorithm for Mining Frequent Closed Itemsets. ACM SIGMOD Workshop on Research Issues in Data Mining and Knowledge Discovery 2000.