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# Peer-to-Peer Networks

Chapter 3: Networks, Searching and Distributed Hash Tables

# Chapter Outline

- Searching and addressing
  - Structured and unstructured networks
- Distributed Hash Tables
  - What they are?
  - How they work?
  - What are they good for?
  - Examples: Chord, CAN, Plaxton/Pastry/Tapestry
- Networks and graphs
  - Graph theory meets networking
  - Different types of graphs and their properties

# Searching and Addressing

- Two basic ways to find objects:

1. Search for them
2. Address them using their unique name

- Both have pros and cons (see below)
- Most existing P2P networks built on searching, but some networks are based on addressing objects
- Difference between searching and addressing is a very fundamental difference
  - Determines how network is constructed
  - Determines how objects are placed
  - "Determines" efficiency of object location
- Let's compare searching and addressing

# Addressing vs. Searching

- "Addressing" networks find objects by addressing them with their unique name (cf. URLs in Web)
- "Searching" networks find objects by searching with keywords that match objects's description (cf. Google)

## Addressing

- Pros:
  - Each object uniquely identifiable
  - Object location can be made efficient
- Cons:
  - Need to know unique name
  - Need to maintain structure required by addresses

## Searching

- Pros:
  - No need to know unique names
    - More user friendly
- Cons:
  - Hard to make efficient
    - Can solve with money, see Google
  - Need to compare actual objects to know if they are same

# Addressing vs. Searching: Examples

| | Searching | Addressing |
|---|---|---|
| Physical name of object | Searching in P2P networks,<br>Searching in filesystem<br>(Desktop searches)<br>(Search components of URL with Google?) | URLs in Web |
| Logical name of object | ?<br>(Search components of URNs) | Object names in DHT,<br>URNs |
| Content or metadata of object | Searching in P2P networks,<br>Standard Google search<br>Desktop searches | N/A |

# Searching, Addressing, and P2P

- We can distinguish two main P2P network types

Unstructured networks/systems

- Based on searching
- Unstructured does NOT mean complete lack of structure
  - Network has graph structure, e.g., scale-free
- Network has structure, but peers are free to join anywhere and objects can be stored anywhere
- So far we have seen unstructured networks

Structured networks/systems

- Based on addressing
- Network structure determines where peers belong in the network and where objects are stored
- How to build structured networks?

# Another Classification of P2P Systems

- Sometimes P2P systems classified in generations
- No 100% consensus on what is in which generation
- 1st generation
  - Typically: Napster
- 2nd generation
  - Typically: Gnutella
- 3rd generation
  - Typically: Superpeer networks
- 4th generation
  - Typically: Distributed hash tables
  - Note: For DHTs, no division into generations yet

# Distributed Hash Tables

- What are they?
- How they work?
- What are they good for?
- Examples:
  - Chord
  - CAN
  - Plaxton/Pastry/Tapestry

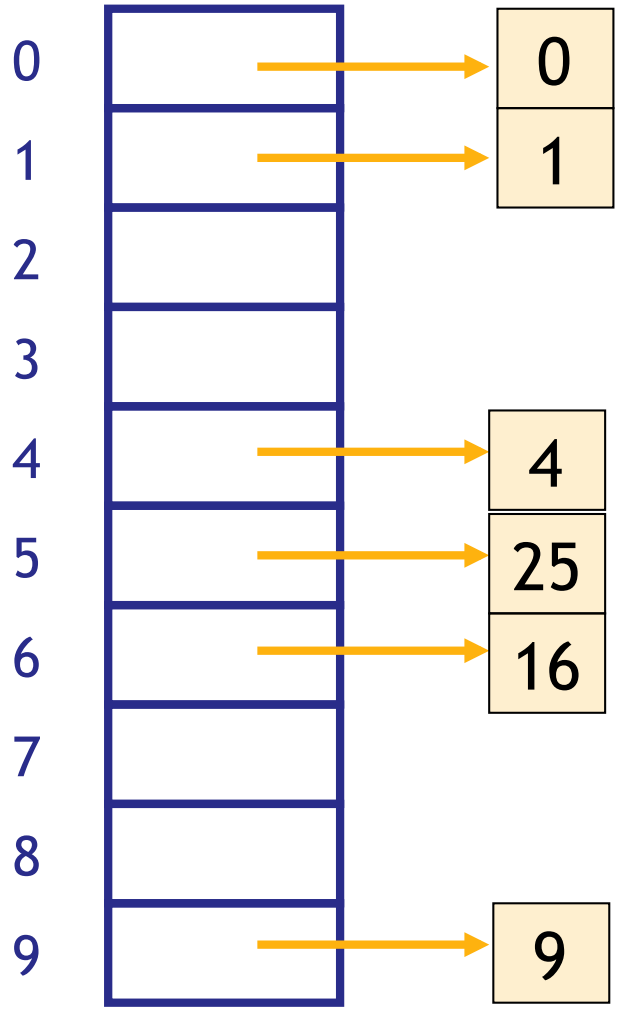
# DHT: Motivation

- Why we need DHTs?
- Searching in P2P networks is not efficient
  - Either centralized system with all its problems
  - Or distributed system with all its problems
  - Hybrid systems cannot guarantee discovery either
- Actual file transfer process in P2P network is scalable
  - File transfers directly between peers
- Searching does not scale in same way
- Original motivation for DHTs: More efficient searching and object location in P2P networks
- Put another way: Use addressing instead of searching

# Recall: Hash Tables

- Hash tables are a well-known data structure
- Hash tables allow insertions, deletions, and finds in constant (average) time
- Hash table is a fixed-size array
  - Elements of array also called *hash buckets*
- *Hash function* maps keys to elements in the array
- Properties of good hash functions:
  - Fast to compute
  - Good distribution of keys into hash table
  - Example: SHA-1 algorithm

# Hash Tables: Example



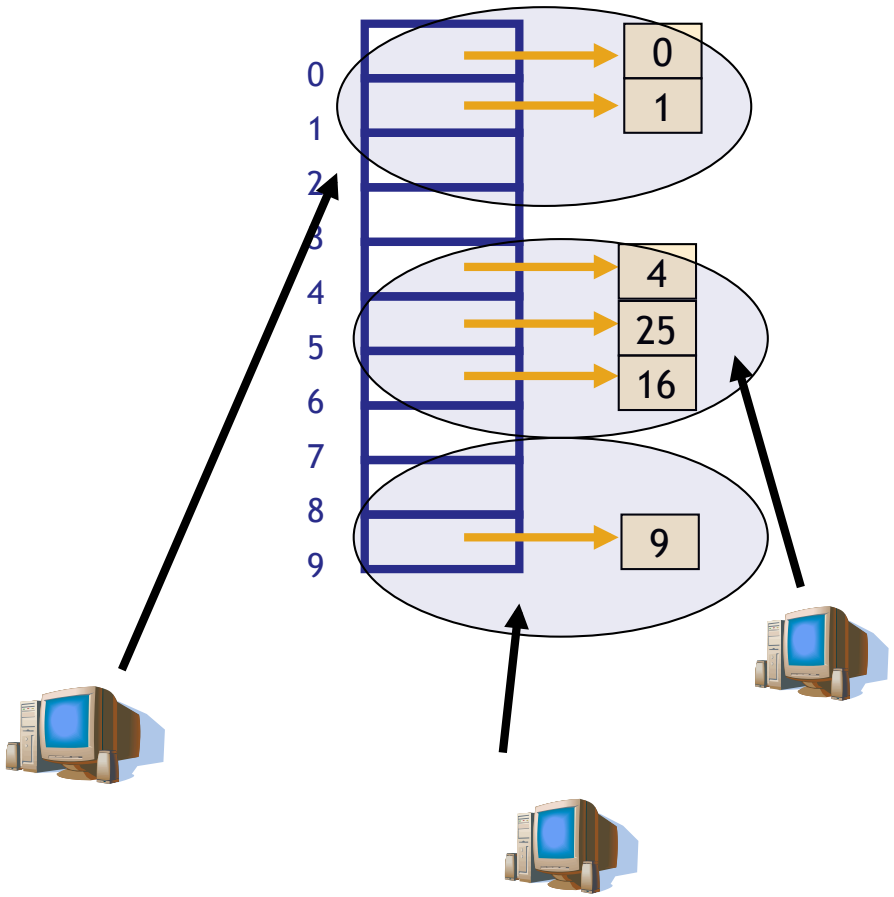

- Hash function:

  *hash(x) = x mod 10*

- Insert numbers 0, 1, 4, 9, 16, and 25
- Easy to find if a given key is present in the table

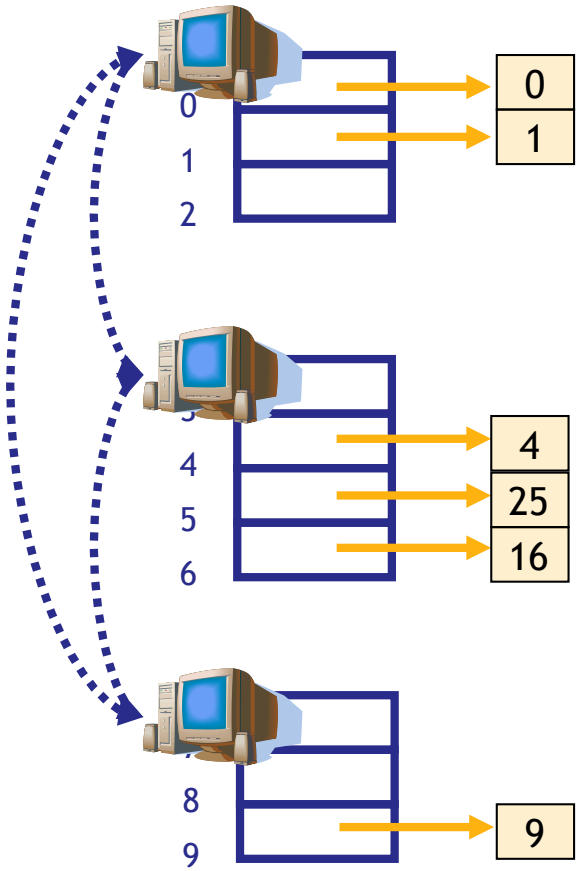
# Distributed Hash Table: Idea

- Hash tables are fast for lookups
- Idea: Distribute hash buckets to peers
- Result is Distributed Hash Table (DHT)
- Need efficient mechanism for finding which peer is responsible for which bucket and routing between them



# DHT: Principle

- In a DHT, each node is responsible for one or more hash buckets
  - As nodes join and leave, the responsibilities change
- Nodes communicate among themselves to find the responsible node
  - Scalable communications make DHTs efficient
- DHTs support all the normal hash table operations



# Summary of DHT Principles

- Hash buckets distributed over nodes
- Nodes form an overlay network
  - Route messages in overlay to find responsible node
- Routing scheme in the overlay network is the difference between different DHTs
- DHT behavior and usage:
  - Node knows “object” name and wants to find it
    - Unique and known object names assumed
  - Node routes a message in overlay to the responsible node
  - Responsible node replies with “object”
    - Semantics of “object” are application defined

# DHT Examples

- In the following look at some example DHTs
  - Chord
  - CAN
  - Tapestry
- Several others exist too
  - Pastry, Plaxton, Kademlia, Koorde, Symphony, P-Grid, CARP, …
- All DHTs provide the same abstraction:
  - DHT stores key-value pairs
  - When given a key, DHT can retrieve/store the value
  - No semantics associated with key or value
- Difference is in overlay routing scheme

# Chord

- Chord was developed at MIT
- Originally published in 2001 at Sigcomm conference

- Chord's overlay routing principle quite easy to understand
  - Paper has mathematical proofs of correctness and performance

- Many projects at MIT around Chord
  - CFS storage system
  - Ivy storage system
  - Plus many others…

# Chord: Basics

- Chord uses SHA-1 hash function
  - Results in a 160-bit object/node identifier
  - Same hash function for objects and nodes
- Node ID hashed from IP address
- Object ID hashed from object name
  - Object names somehow assumed to be known by everyone

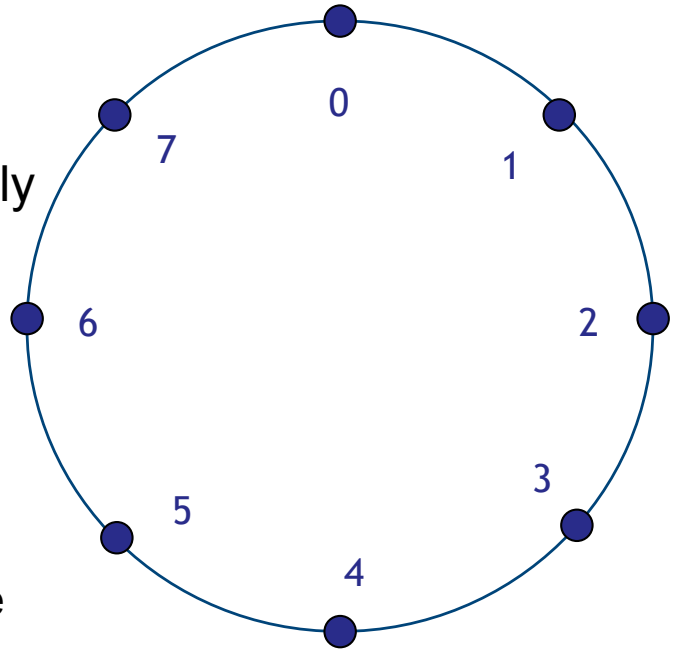
- SHA-1 gives a 160-bit identifier space
- Organized in a ring which wraps around
  - Nodes keep track of predecessor and successor
  - Node responsible for objects between its predecessor and itself
  - Overlay is often called “Chord ring” or “Chord circle”

# Chord: Examples

- Below examples for:
  - How to join the Chord ring
  - How to store and retrieve values

# Joining: Step-By-Step Example

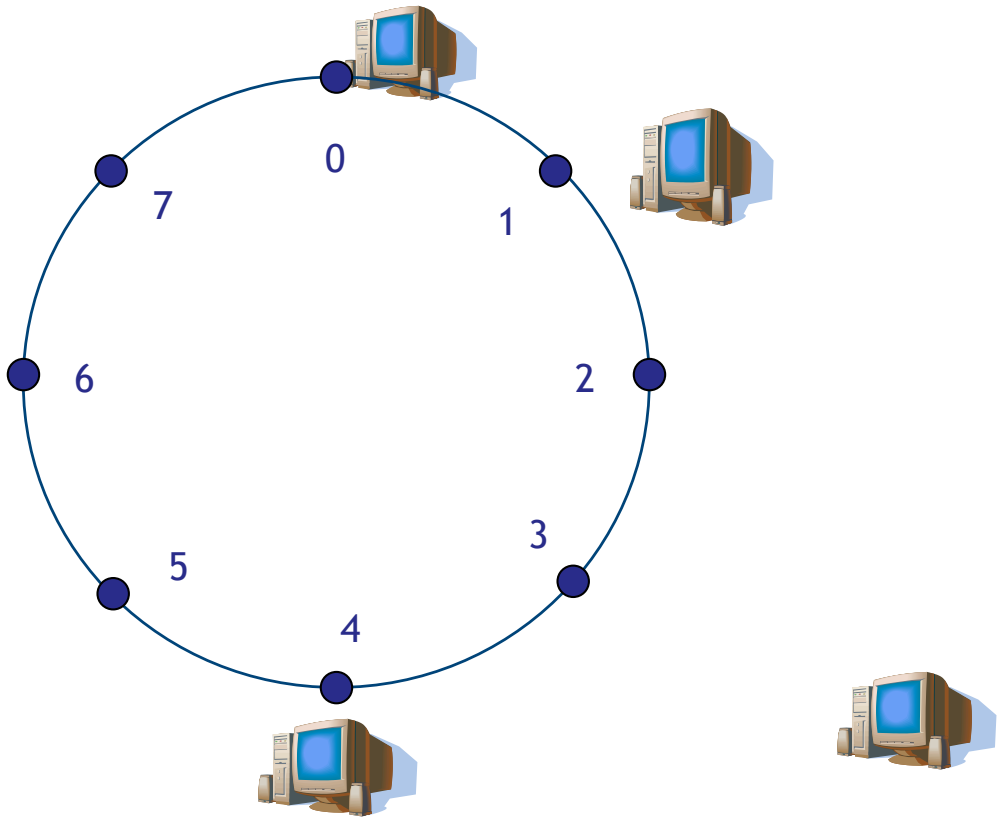
- Setup: Existing network with nodes on 0, 1 and 4
- Note: Protocol messages simply examples
- Many different ways to implement Chord
  - Here only conceptual example
  - Covers all important aspects



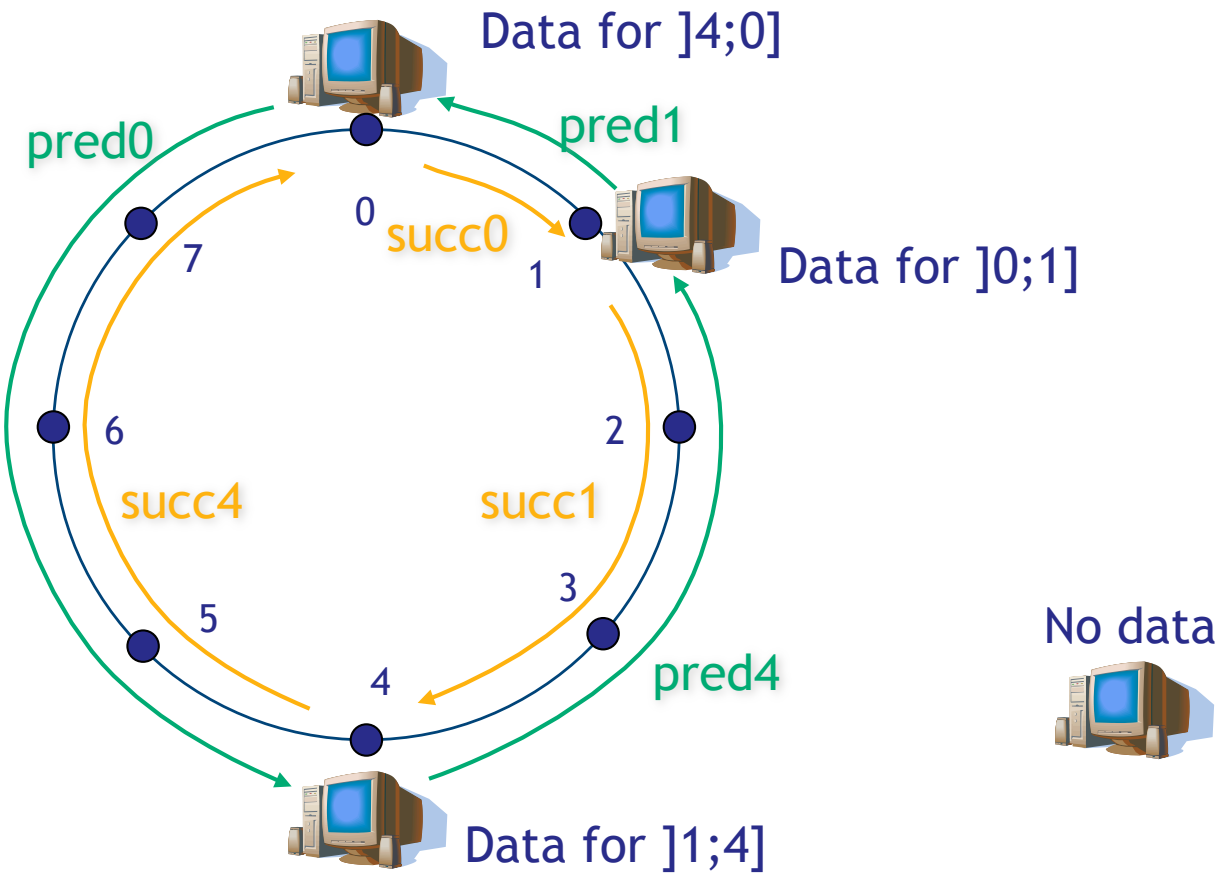
# Joining: Step-By-Step Example: Start

- New node wants to join
- Hash of the new node: 6
- Known node in network: Node1

- Contact Node1
  - Include own hash

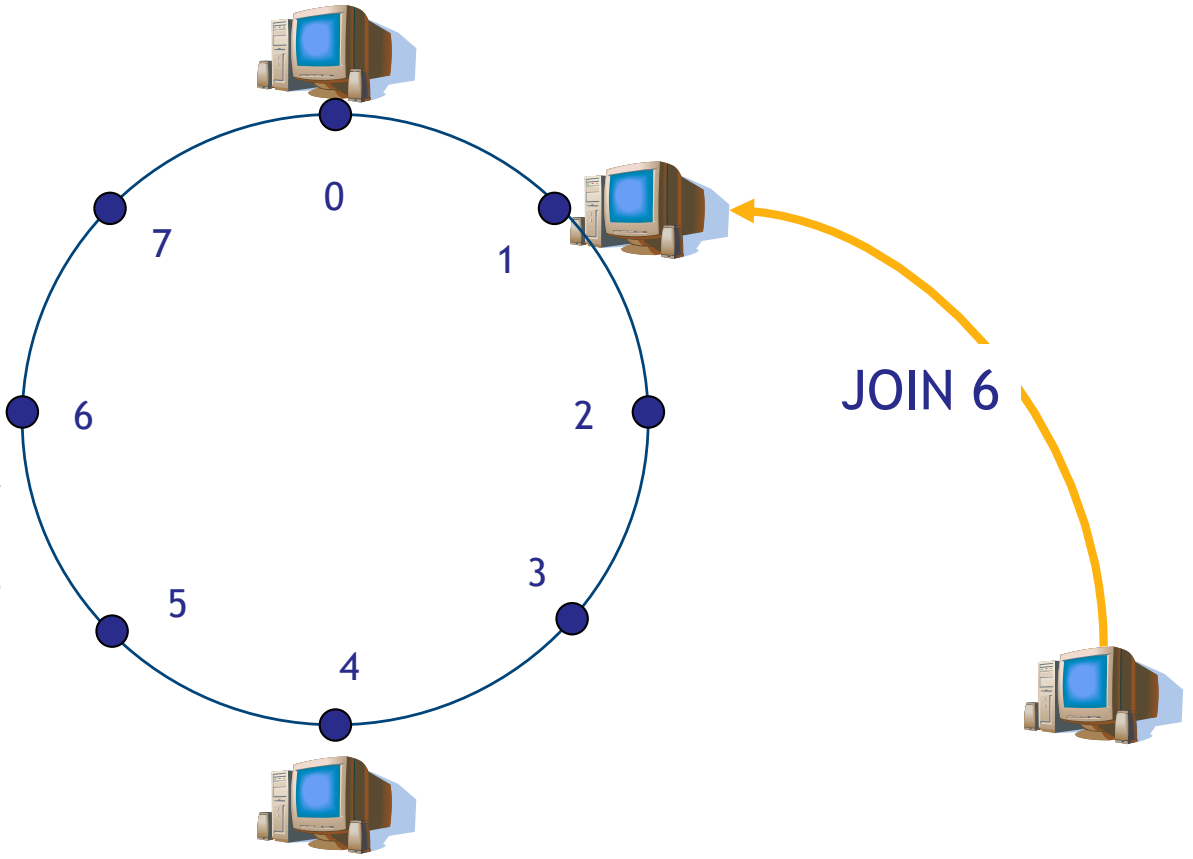


# Joining: Step-By-Step Example: Situation Before Join

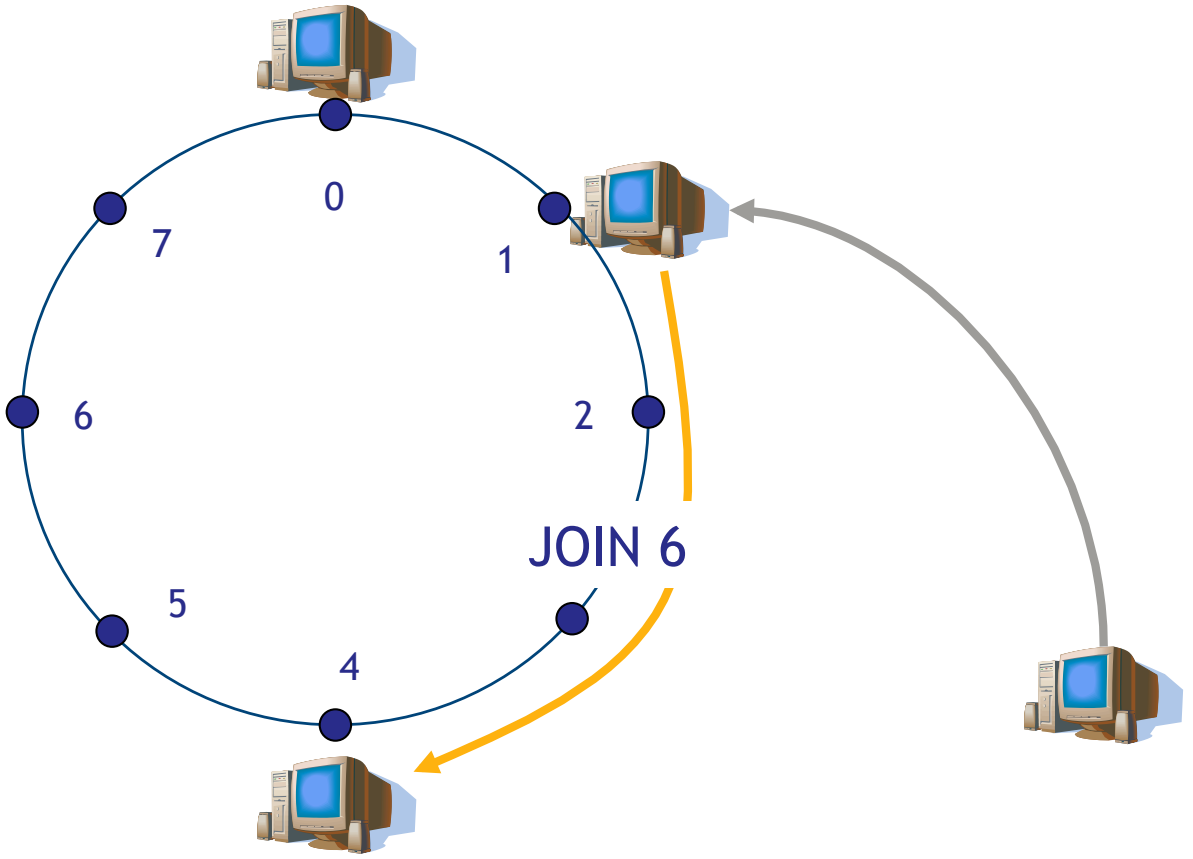


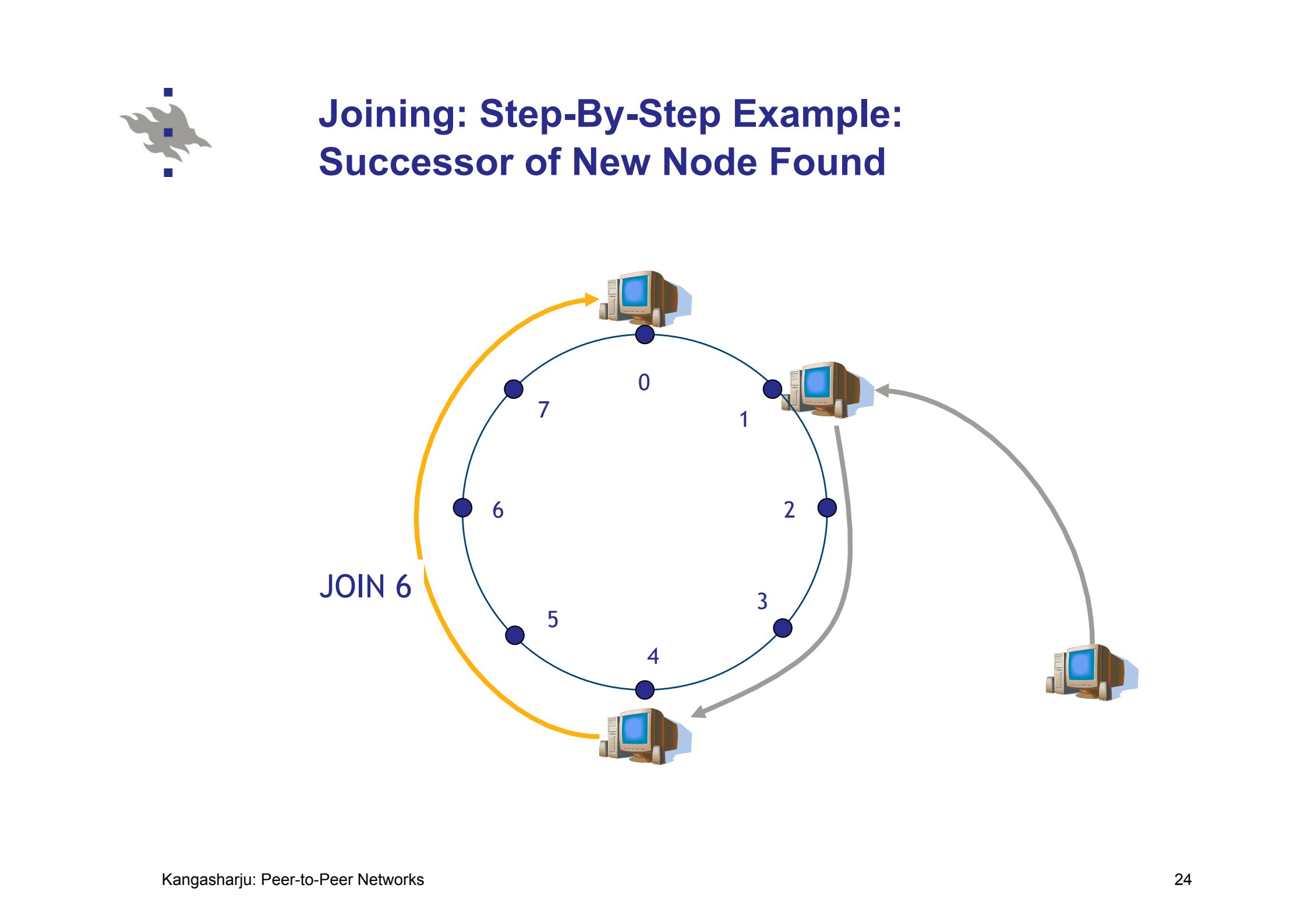
# Joining: Step-By-Step Example: Contact known node

- Arrows indicate open connections
- Example assumes connections are kept open, i.e., messages processed recursively
- Iterative processing is also possible



# Joining: Step-By-Step Example: Join gets routed along the network





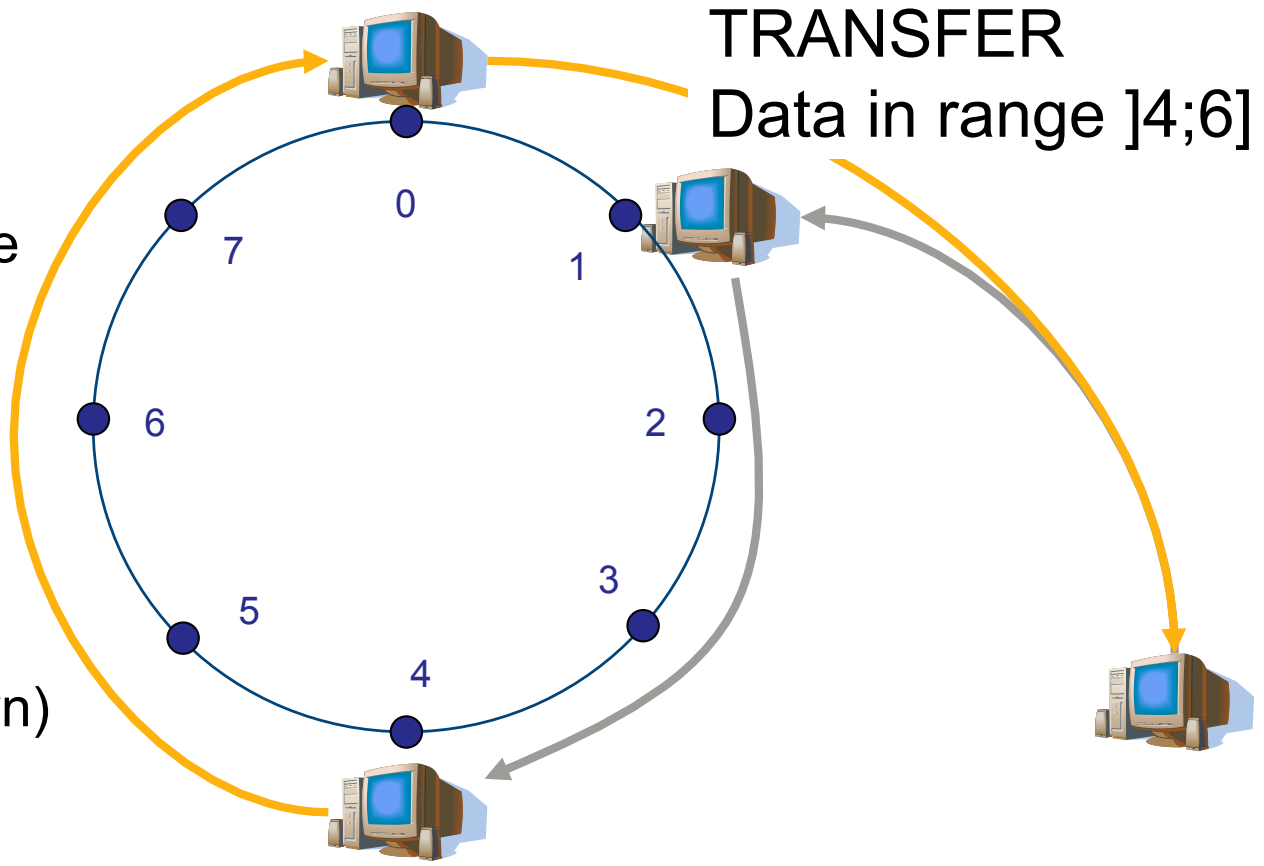
# Joining: Step-By-Step Example: Successor of New Node Found

# Joining: Step-By-Step Example: Joining Successful + Transfer

Joining is successful

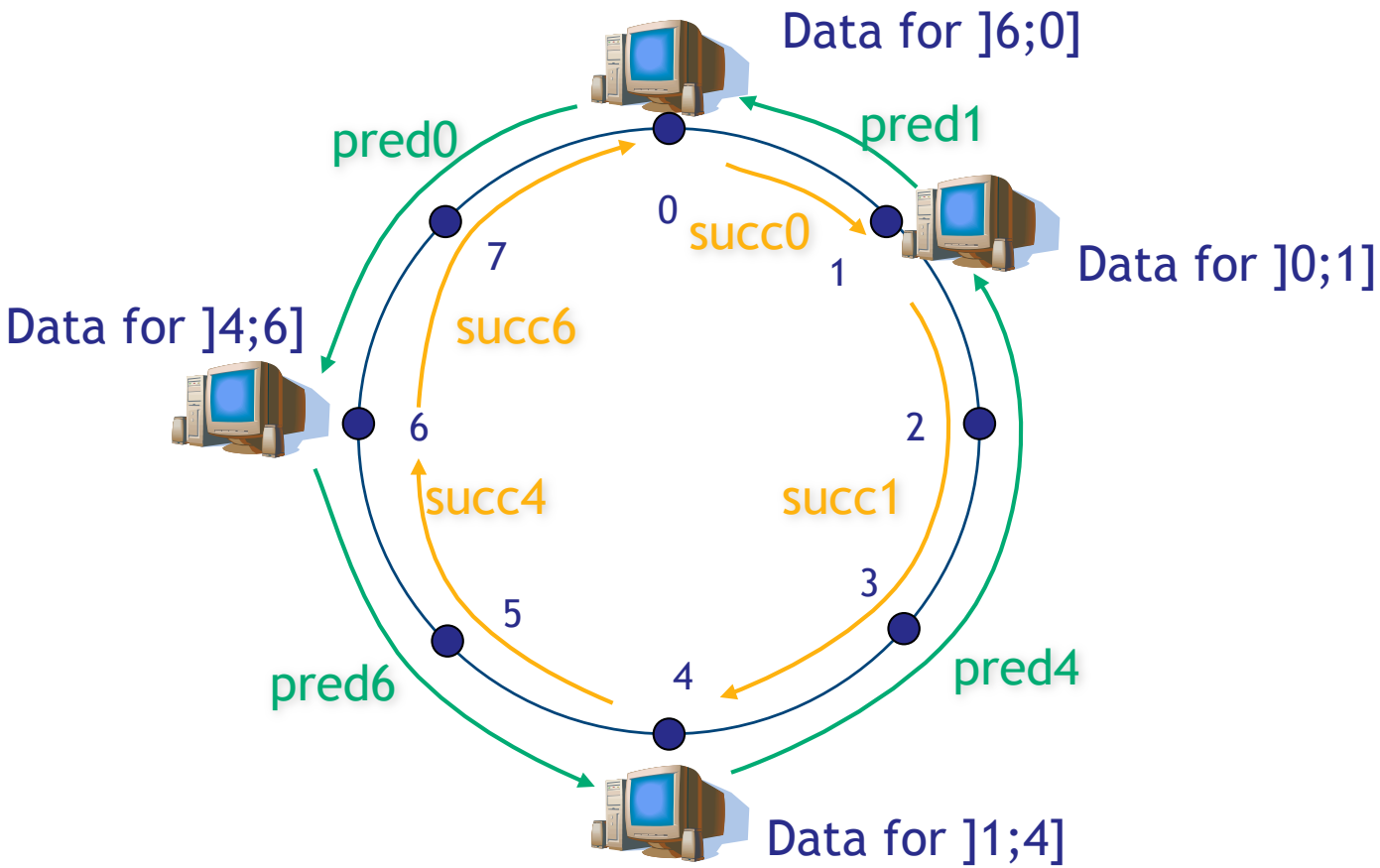
Old responsible node transfers data that should be in new node

New node informs Node4 about new successor (not shown)



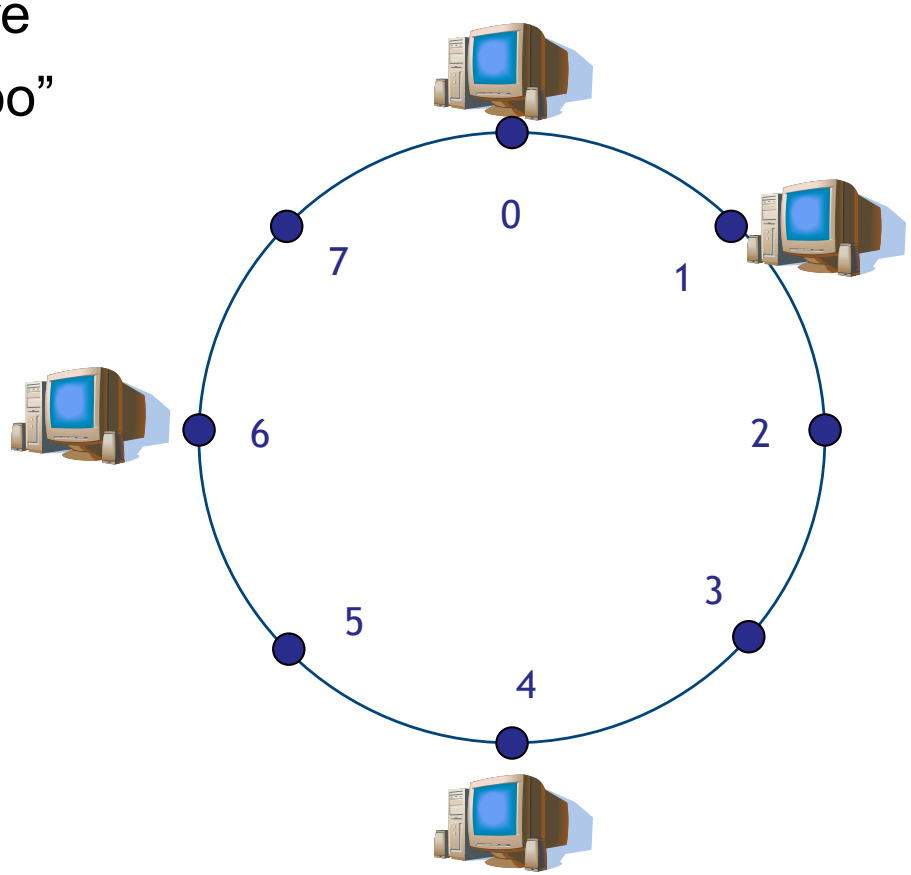

Note: Transferring can happen also later

# Joining: Step-By-Step Example: All Is Done

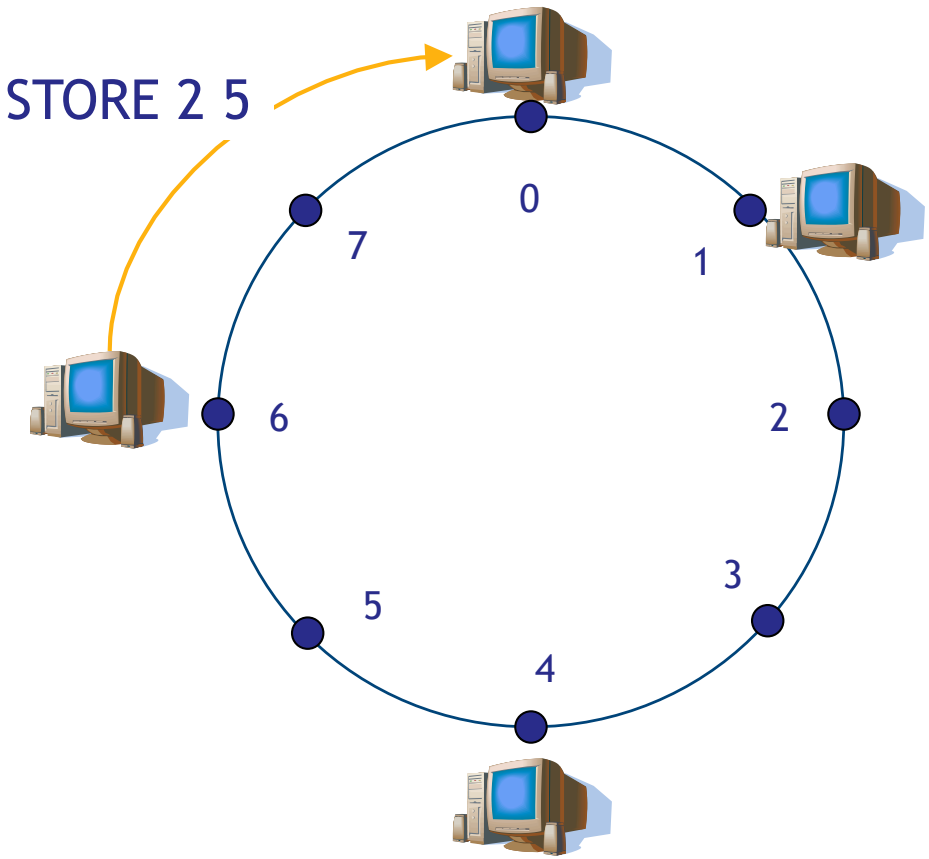


# Storing a Value

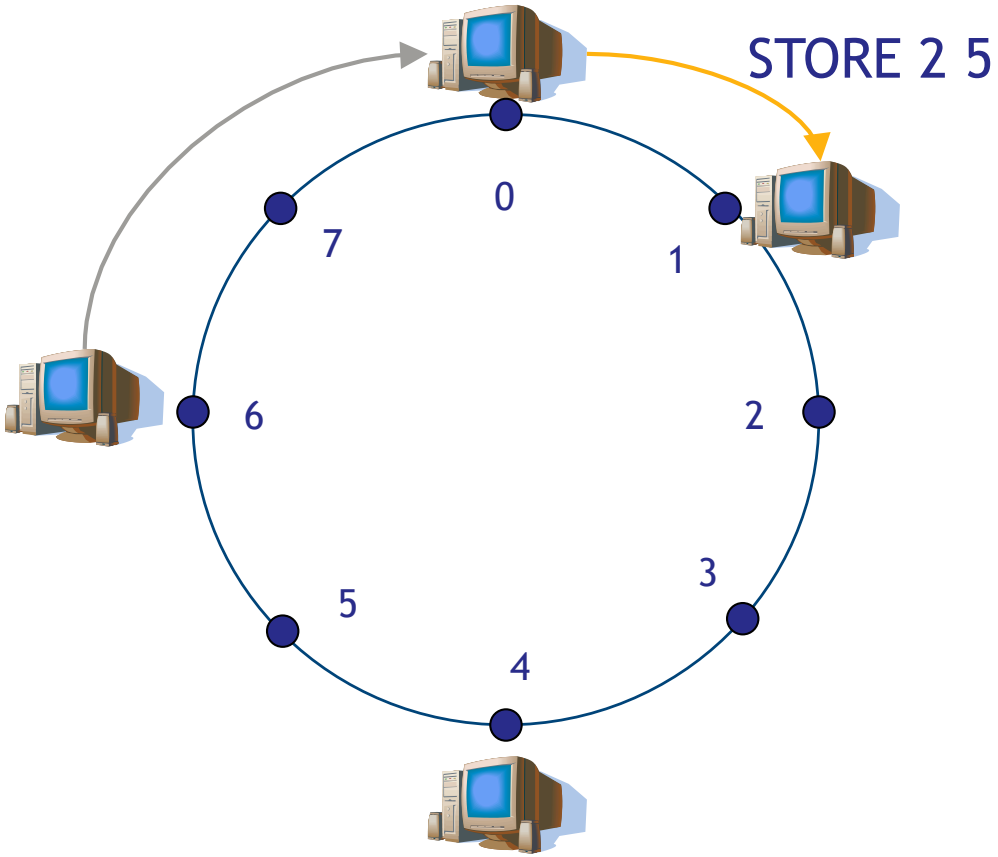
- Node 6 wants to store object with name “Foo” and value 5
- hash(Foo) = 2



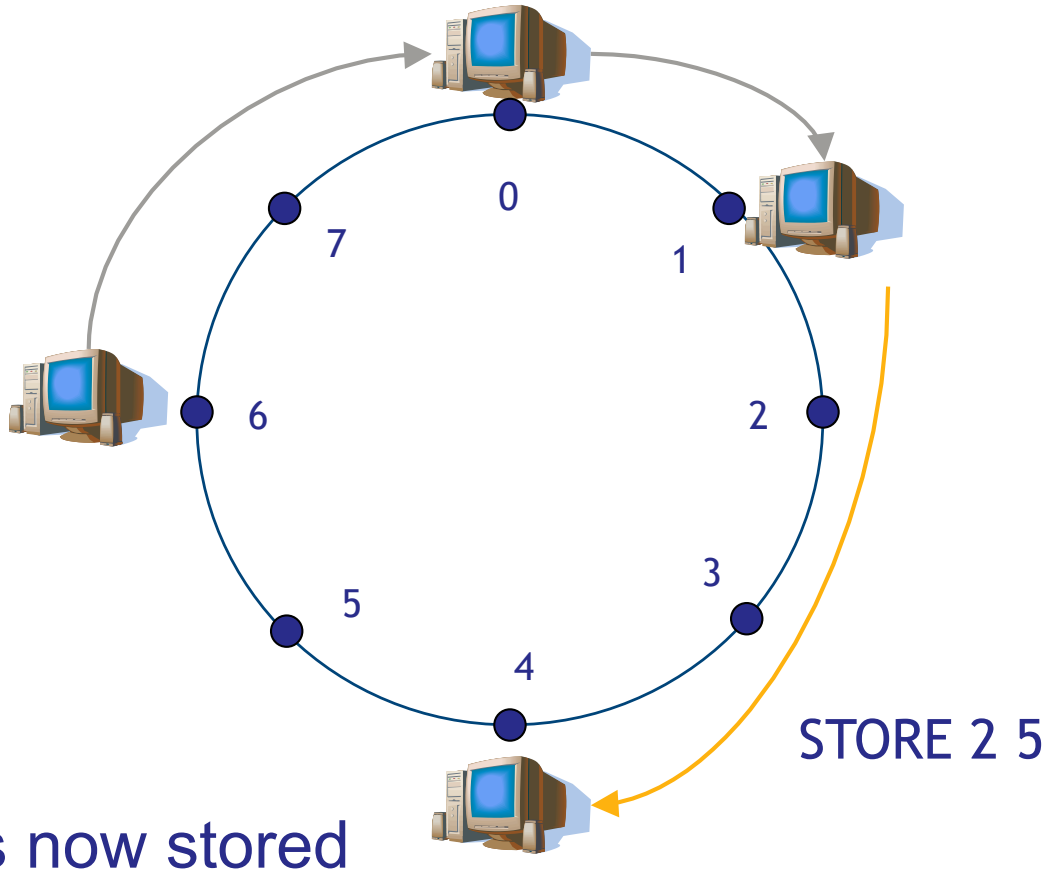
# Storing a Value



# Storing a Value



# Storing a Value

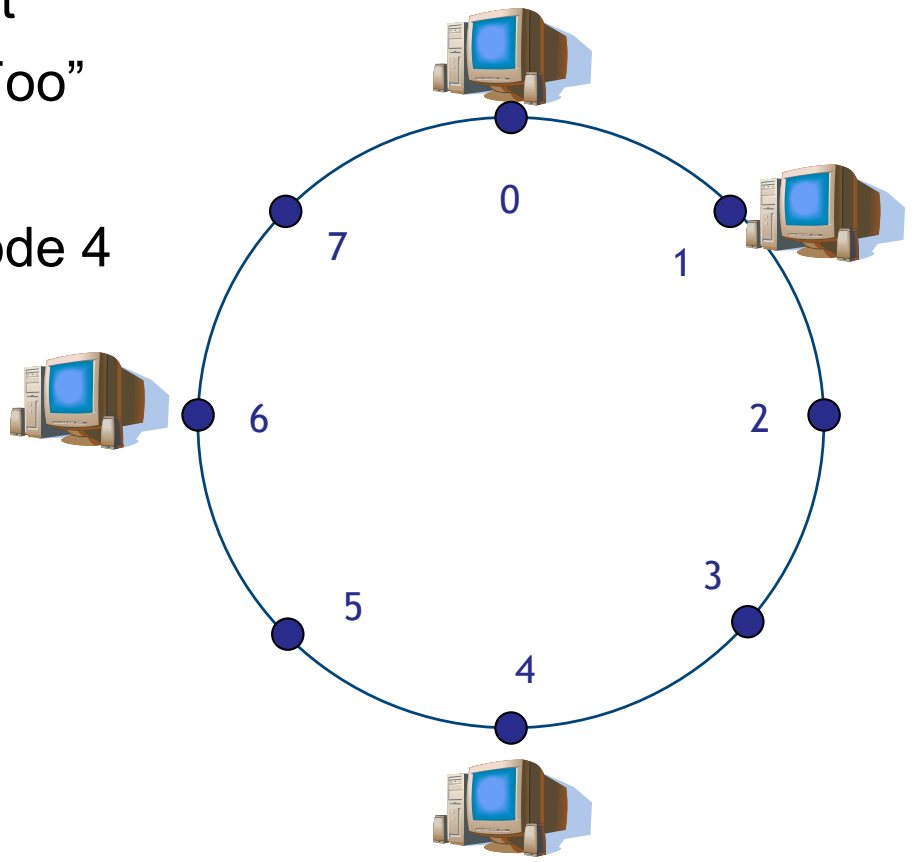



Value is now stored in node 4.

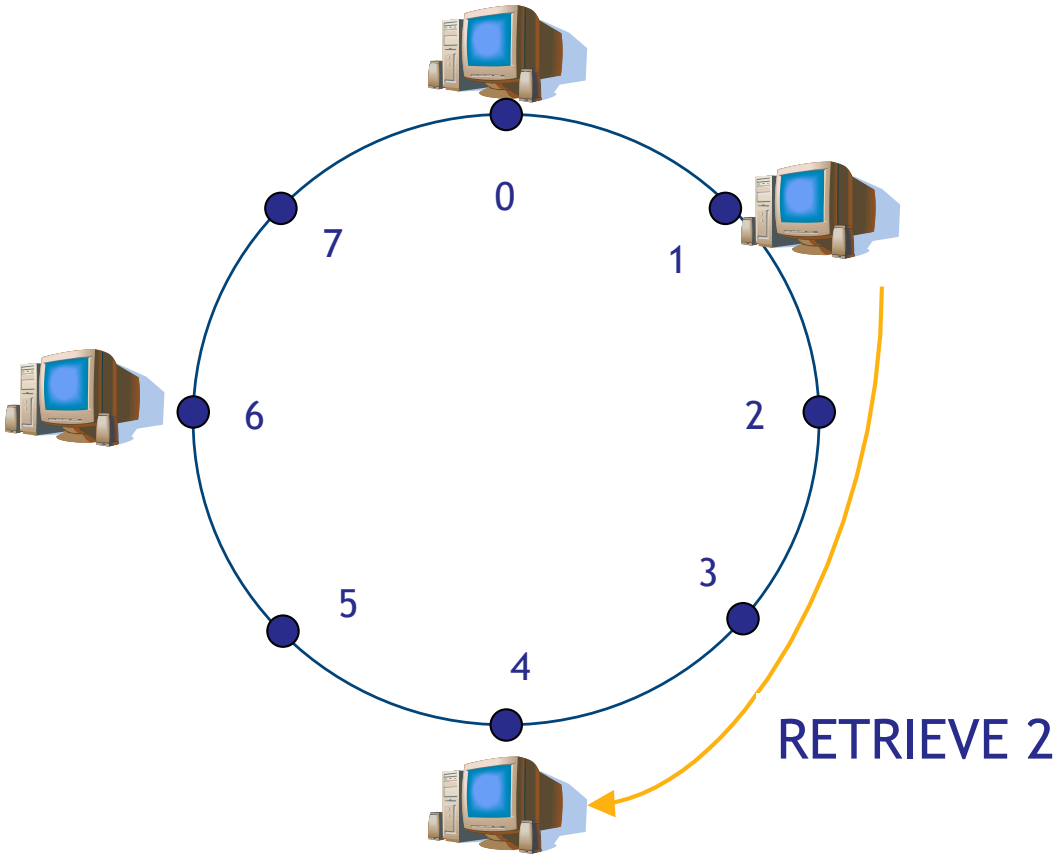
# Retrieving a Value

- Node 1 wants to get object with name “Foo”
- hash(Foo) = 2

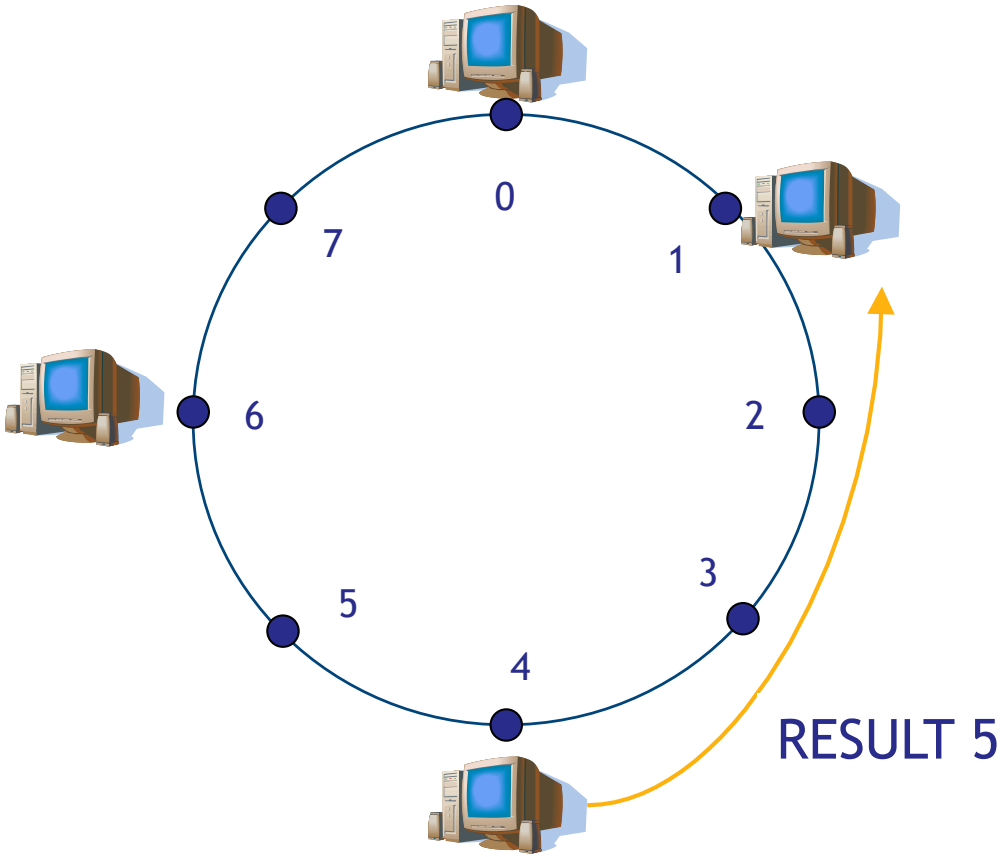
→ Foo is stored on node 4



# Retrieving a Value



# Retrieving a Value



# Chord: Scalable Routing

- Routing happens by passing message to successor
- What happens when there are 1 million nodes?
  - On average, need to route 1/2-way across the ring
  - In other words, 0.5 million hops! Complexity *O(n)*
- How to make routing scalable?
- Answer: Finger tables
- Basic Chord keeps track of predecessor and successor
- Finger tables keep track of more nodes
  - Allow for faster routing by jumping long way across the ring
  - Routing scales well, but need more state information
- Finger tables not needed for correctness, only performance improvement

# Chord: Finger Tables

- In *m*-bit identifier space, node has up to *m* fingers
- Fingers are stored in the finger table

- Row *i* in finger table at node *n* contains first node *s* that succeeds *n* by at least $2^{i-1}$ on the ring
- In other words:

$$finger[i] = successor(n + 2^{i-1})$$

- First finger is the successor
- Distance to *finger[i]* is at least $2^{i-1}$

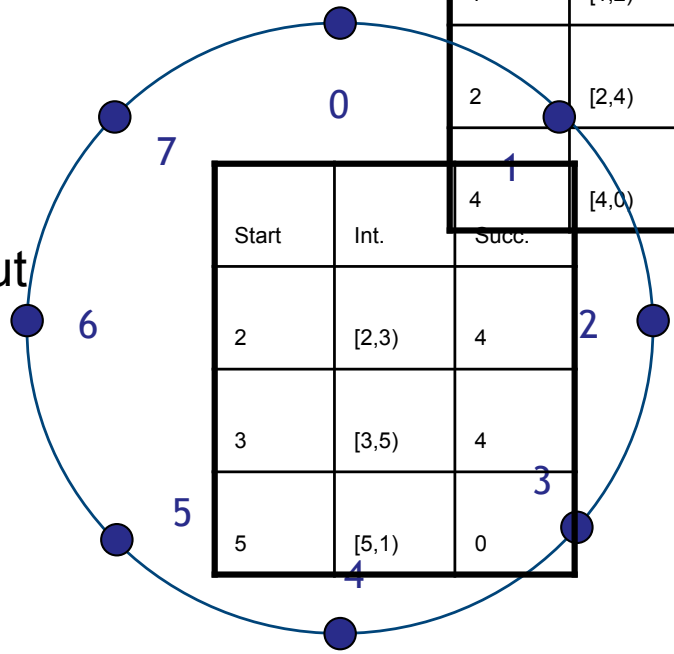
# Chord: Scalable Routing

- Finger intervals increase with distance from node n
  - If close, short hops and if far, long hops

Two key properties:

- Each node only stores information about a small number of nodes
- Cannot in general determine the successor of an arbitrary ID

- Example has three nodes at 0, 1, and 4
- 3-bit ID space --> 3 rows of fingers




| Start | Int. | Succ. |
|---|---|---|
| 1 | [1,2) | 1 |
| 2 | [2,4) | 4 |
| 4 | [4,0) | 4 |

| Start | Int. | Succ. |
|---|---|---|
| 2 | [2,3) | 4 |
| 3 | [3,5) | 4 |
| 5 | [5,1) | 0 |

| Start | Int. | Succ. |
|---|---|---|
| 5 | [5,6) | 0 |
| 6 | [6,0) | 0 |

# Chord: Performance

- Search performance of "pure" Chord *O(n)*
  - Number of nodes is *n*
- With finger tables, need *O(log n)* hops to find the correct node
  - Fingers separated by at least $2^{i-1}$
  - With high probability, distance to target halves at each step
  - In beginning, distance is at most $2^m$
  - Hence, we need at most *m* hops
- For state information, "pure" Chord has only successor and predecessor, *O(1)* state
- For finger tables, need *m* entries
  - Actually, only *O(log n)* are distinct
  - Proof is in the paper

# CAN: Content Addressable Network

- CAN developed at UC Berkeley
- Originally published in 2001 at Sigcomm conference(!)

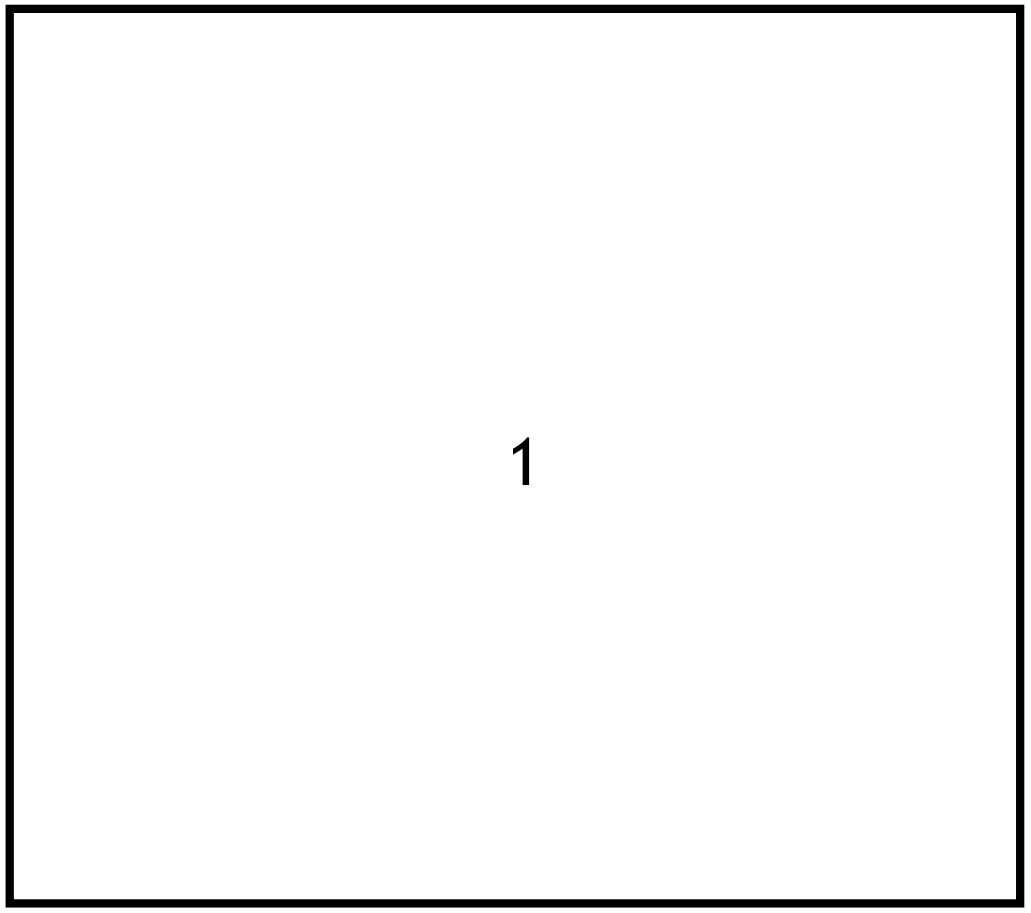
- CANs overlay routing easy to understand
  - Paper concentrates more on performance evaluation
  - Also discussion on how to improve performance by tweaking

- CAN project did not have much of a follow-up
  - Only overlay was developed, no bigger follow-ups

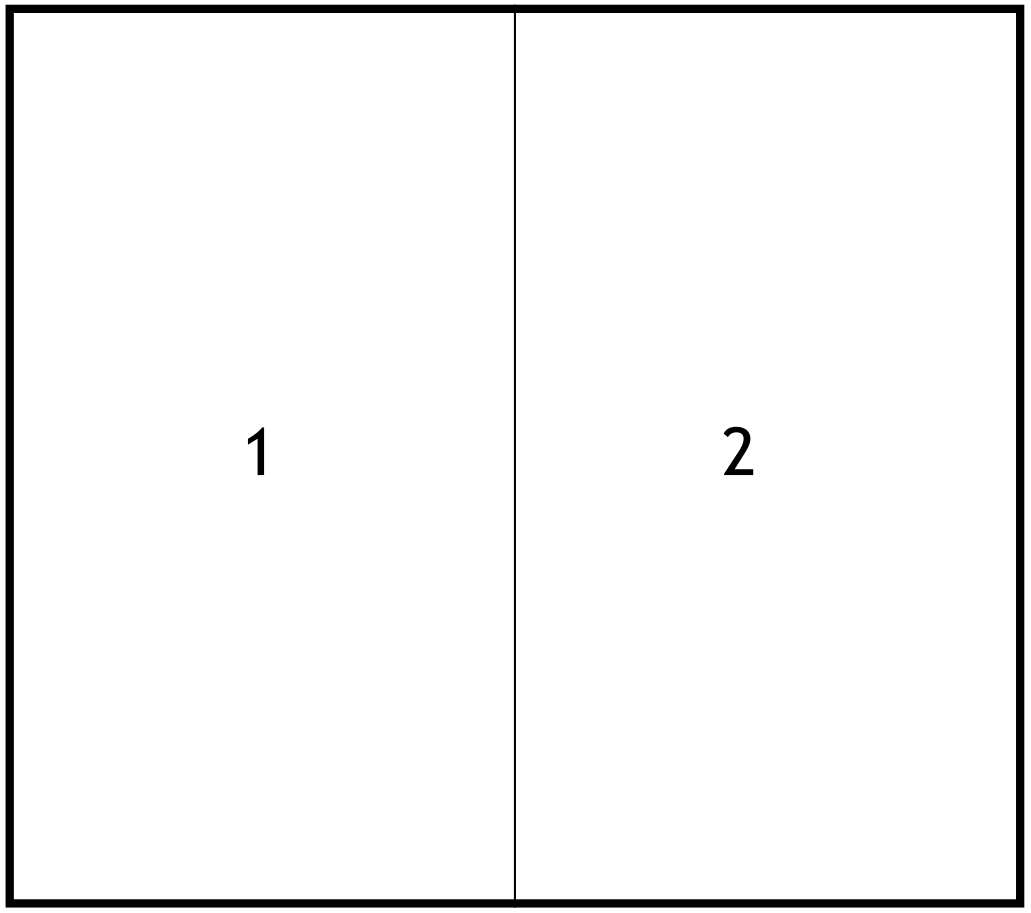
# CAN: Basics

- CAN based on N-dimensional Cartesian coordinate space
  - Our examples: N = 2
  - One hash function for each dimension
- Entire space is partitioned amongst all the nodes
  - Each node owns a zone in the overall space
- Abstractions provided by CAN:
  - Can store data at points in the space
  - Can route from one point to another
- Point = Node that owns the zone in which the point (coordinates) is located
- Order in which nodes join is important

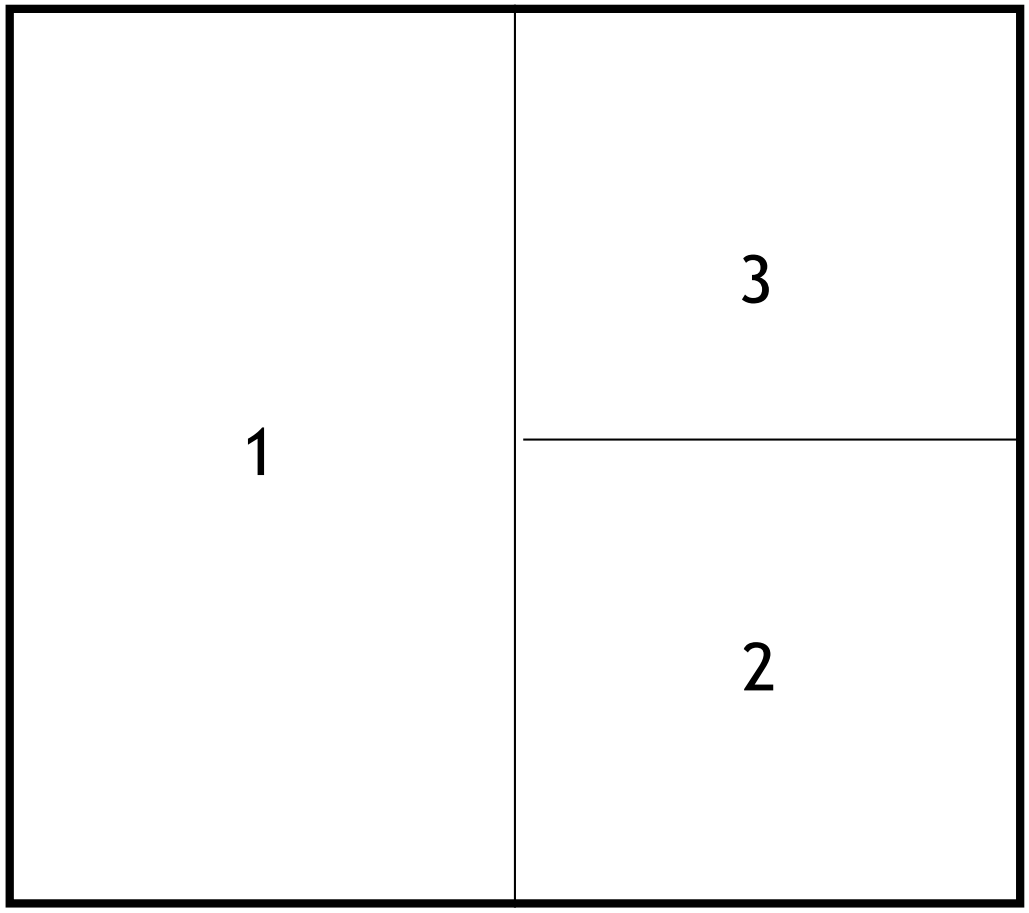
# CAN: Partitioning



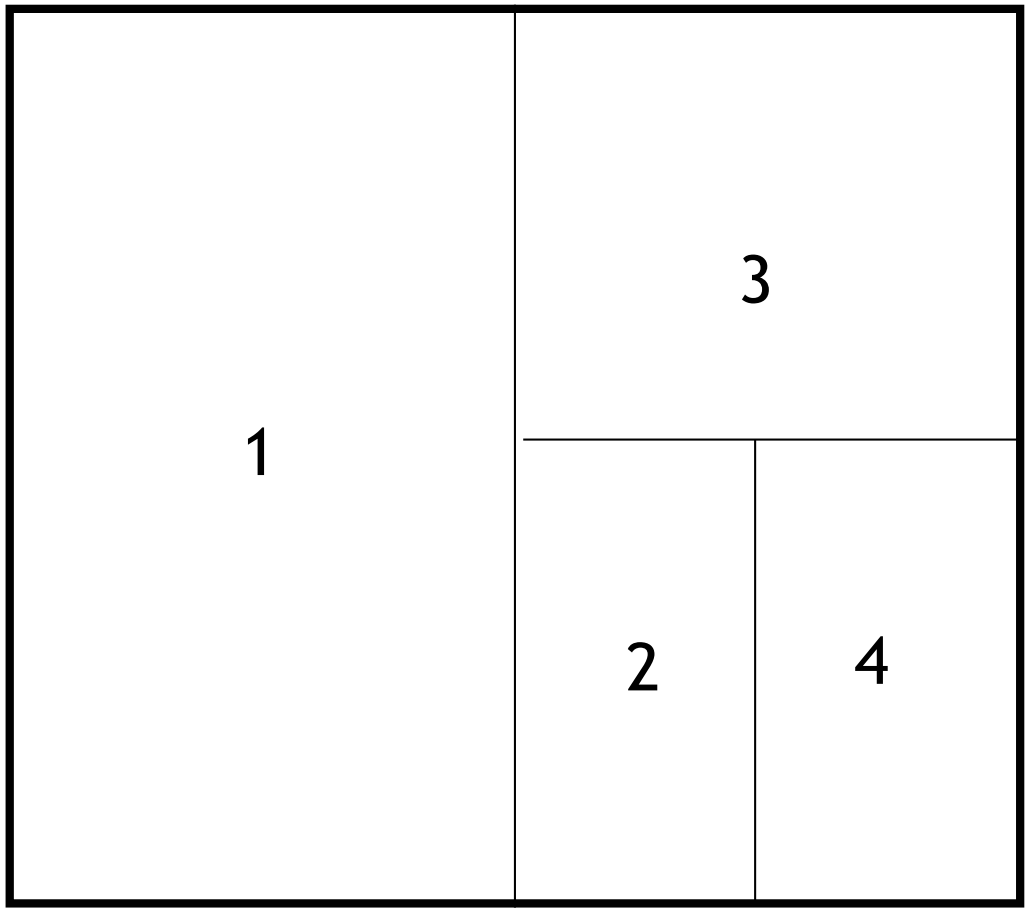
# CAN: Partitioning



# CAN: Partitioning

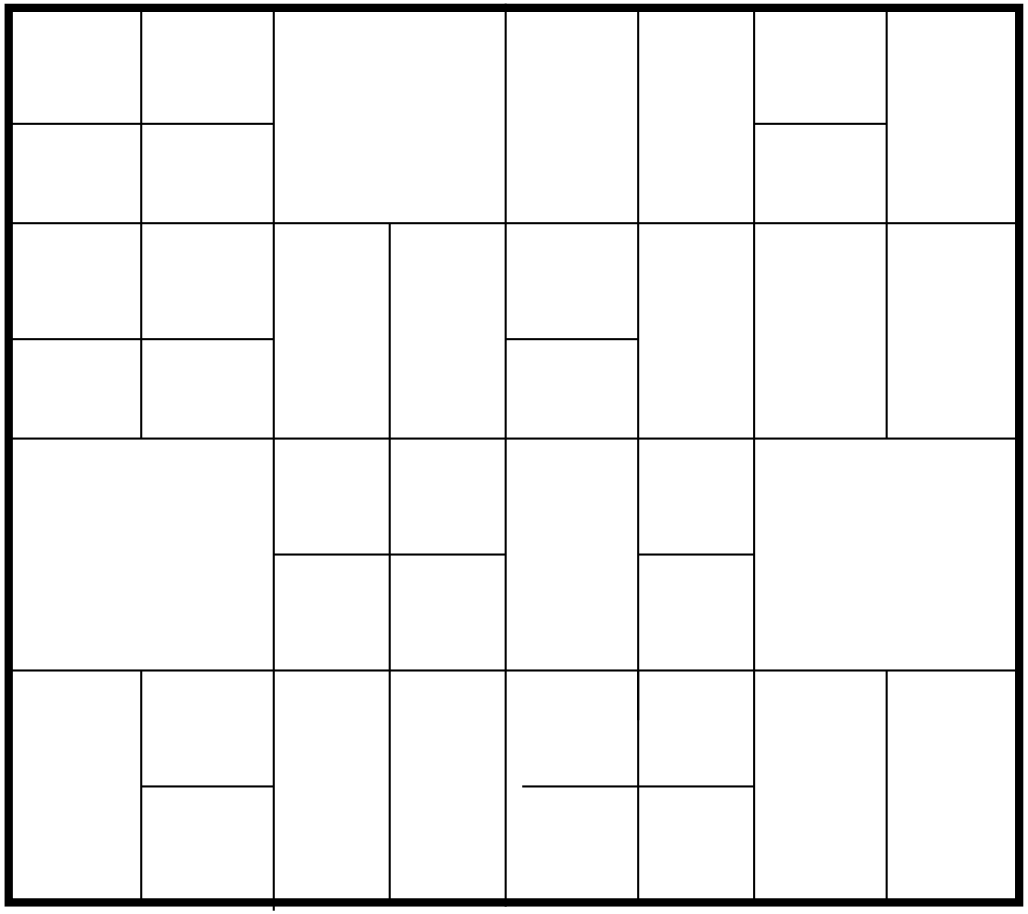


# CAN: Partitioning



# CAN: Partitioning

- CAN forms a d-dimensional torus

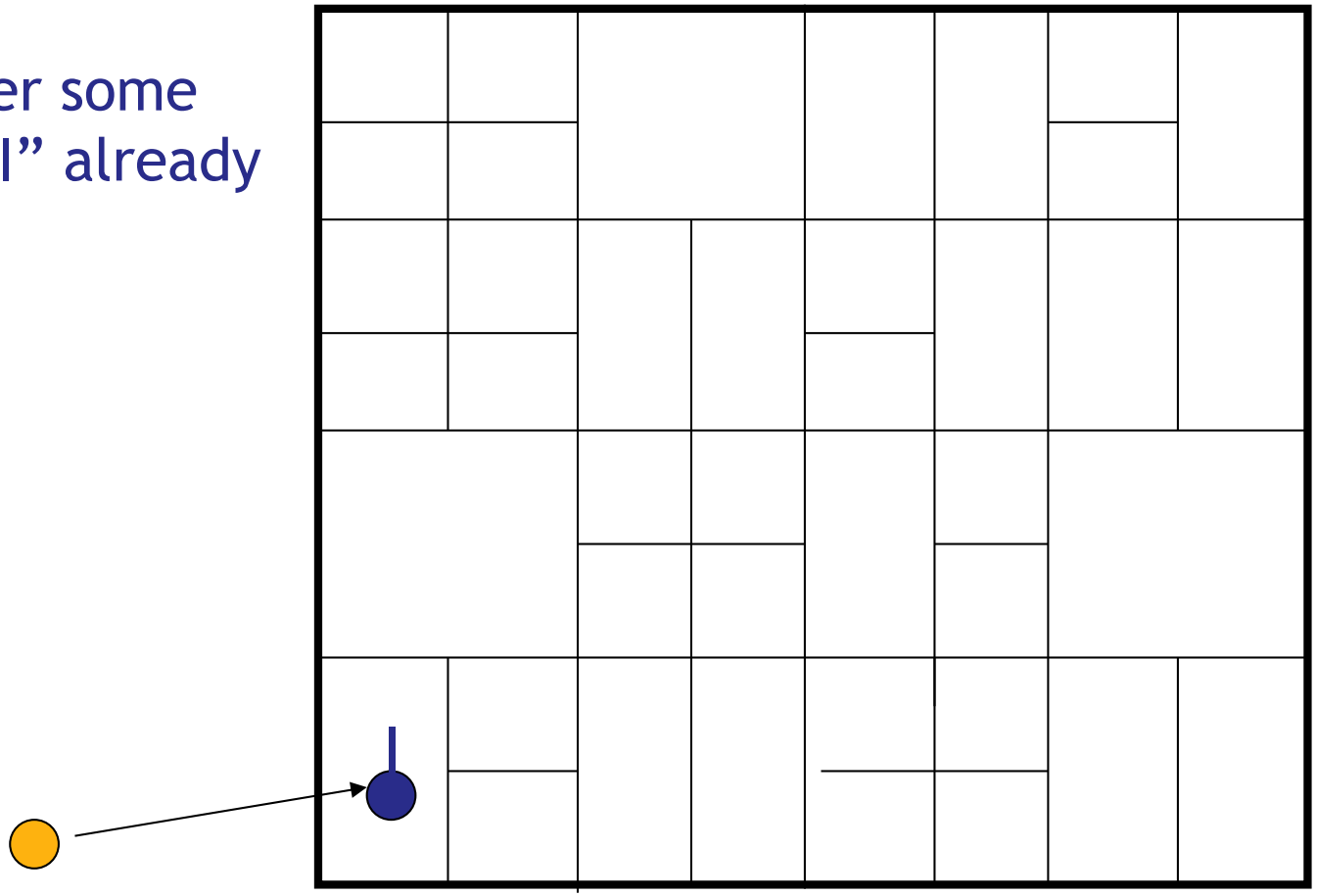


# CAN: Examples

- Below examples for:
  - How to join the network
  - How routing tables are managed
  - How to store and retrieve values

# CAN: Node Insertion

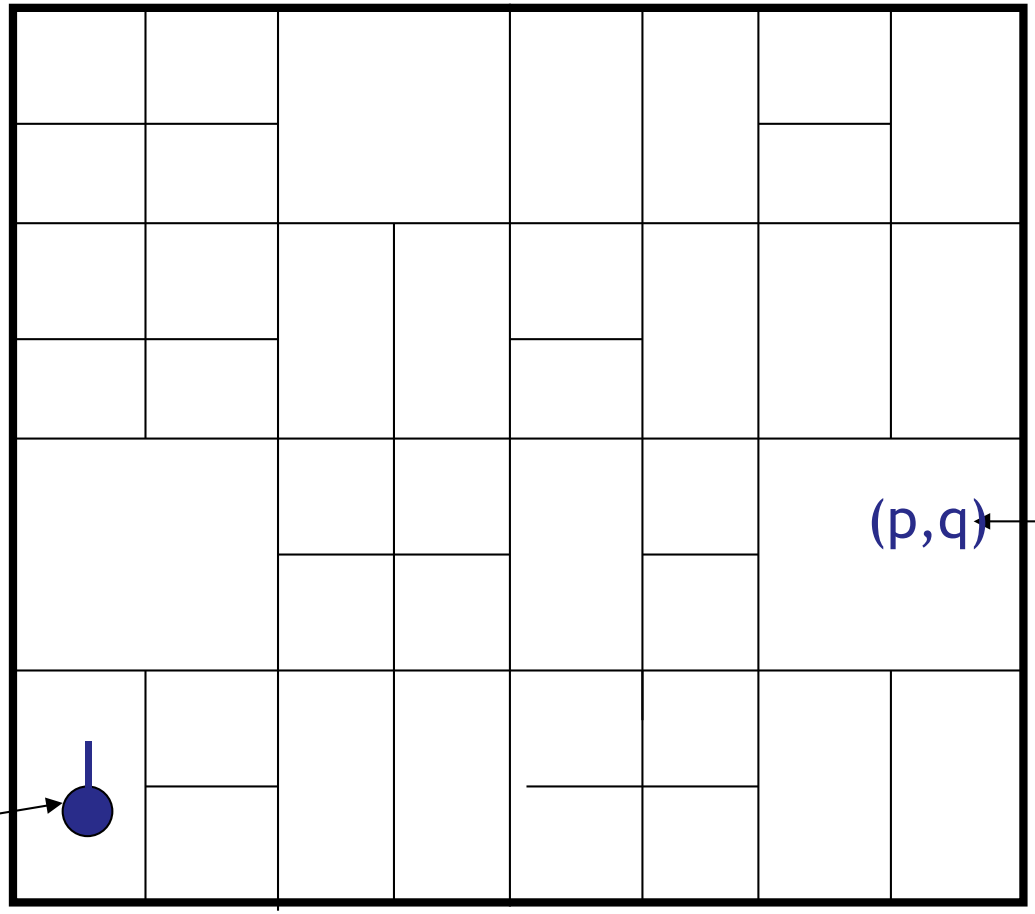
Discover some node “I” already in CAN



New node

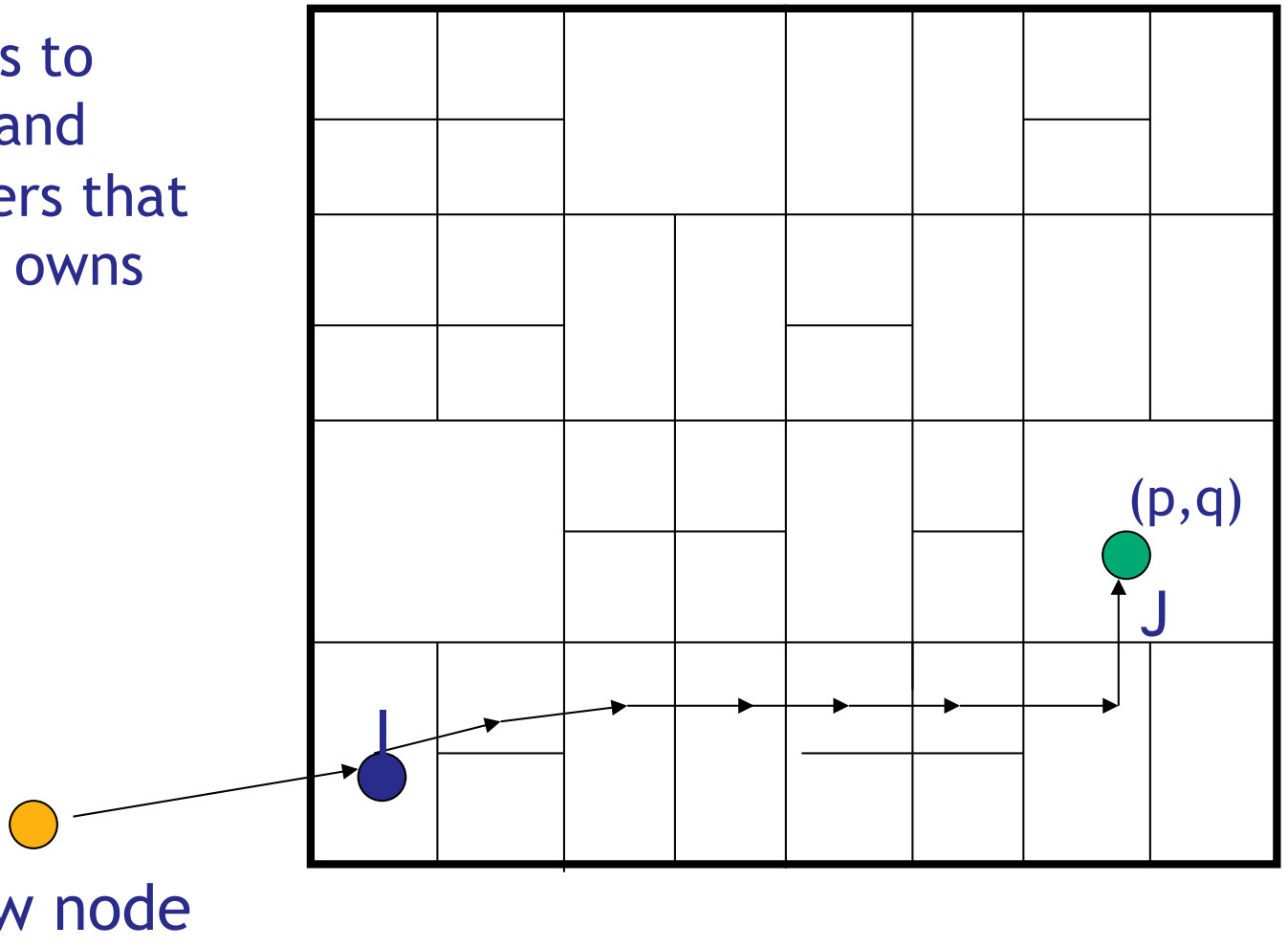
# CAN: Node Insertion

New node picks its coordinates in space



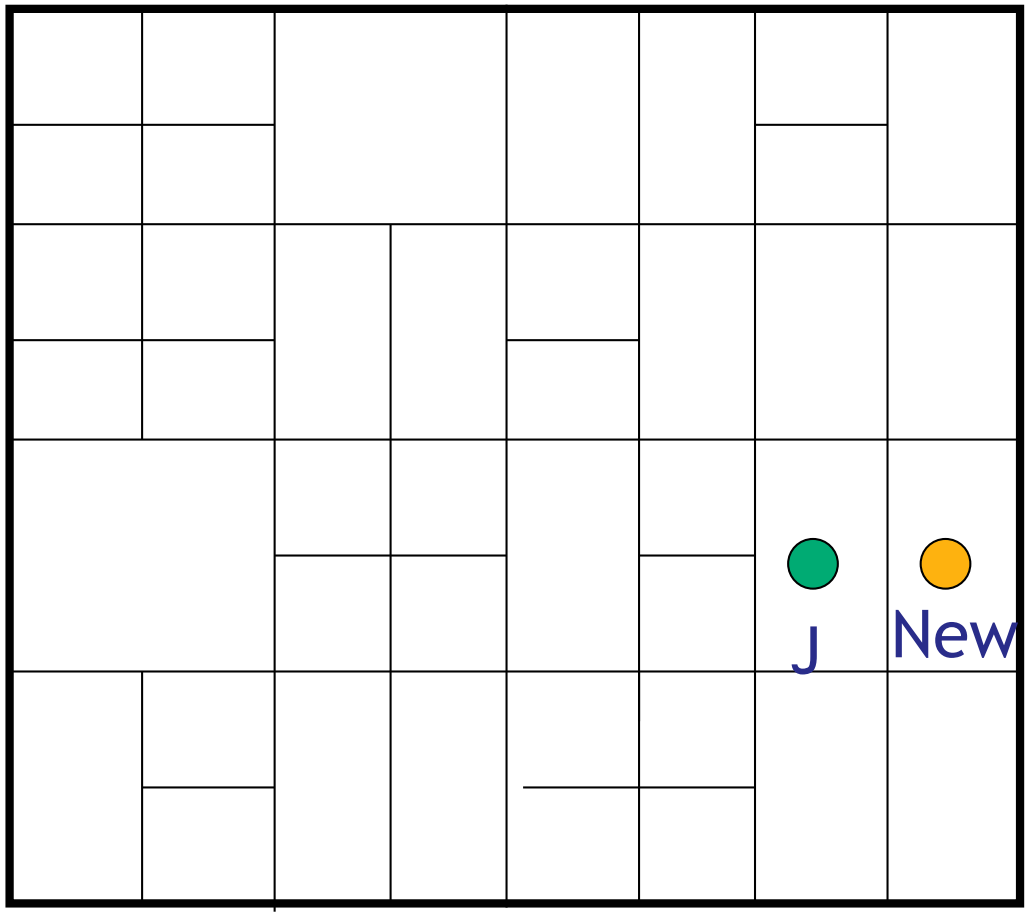
# CAN: Node Insertion

I routes to (p,q), and discovers that node J owns (p,q)

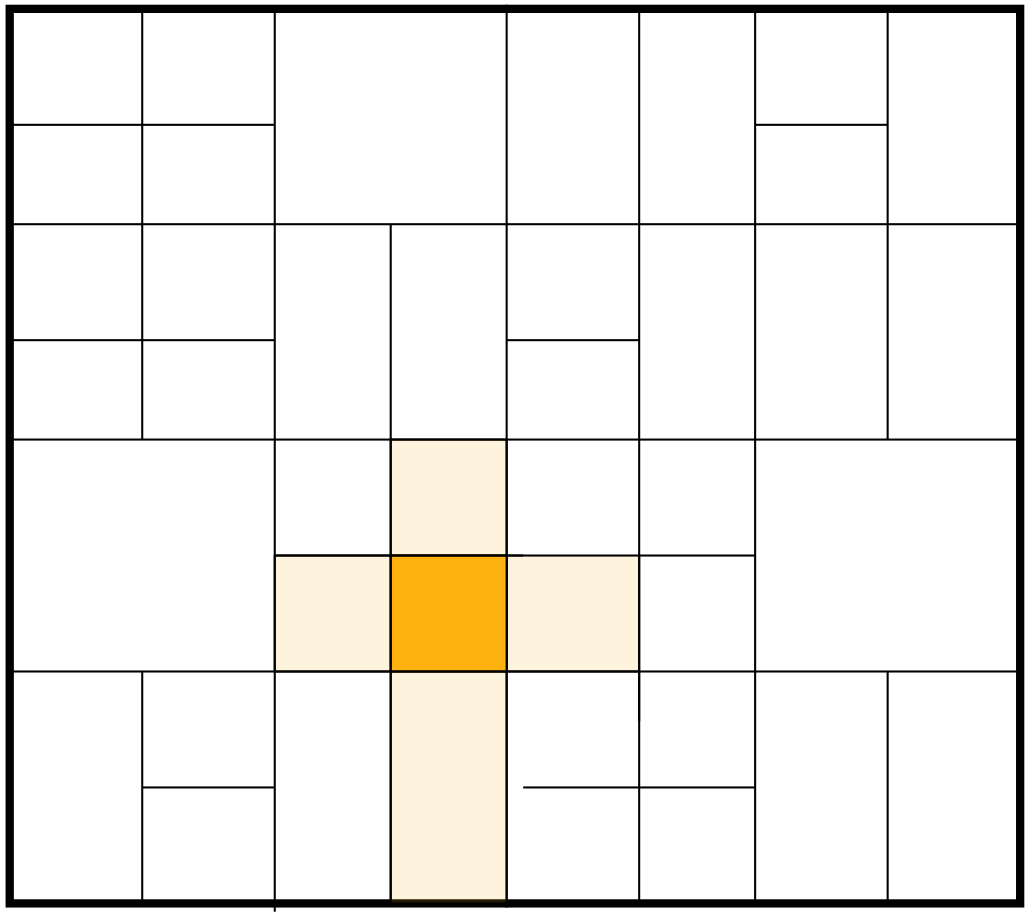


# CAN: Node Insertion

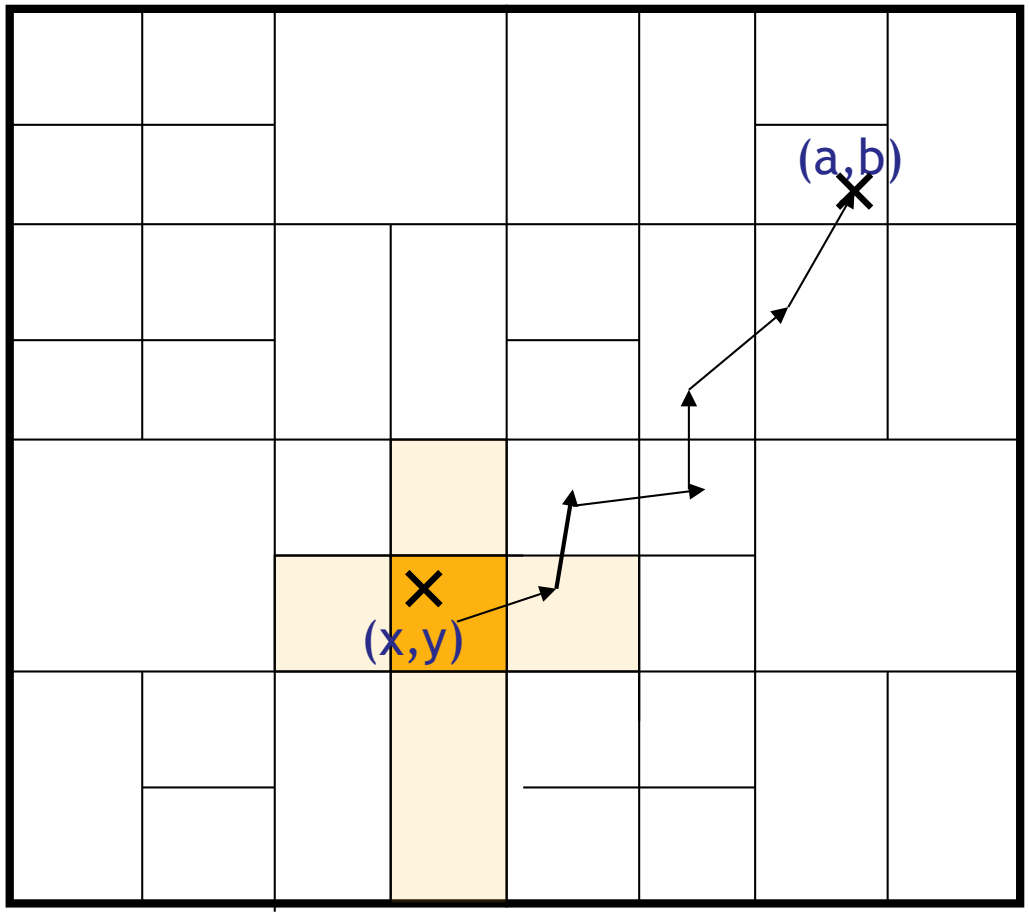
Split J's zone in half. New owns one half



# CAN: Routing Table



# CAN: Routing

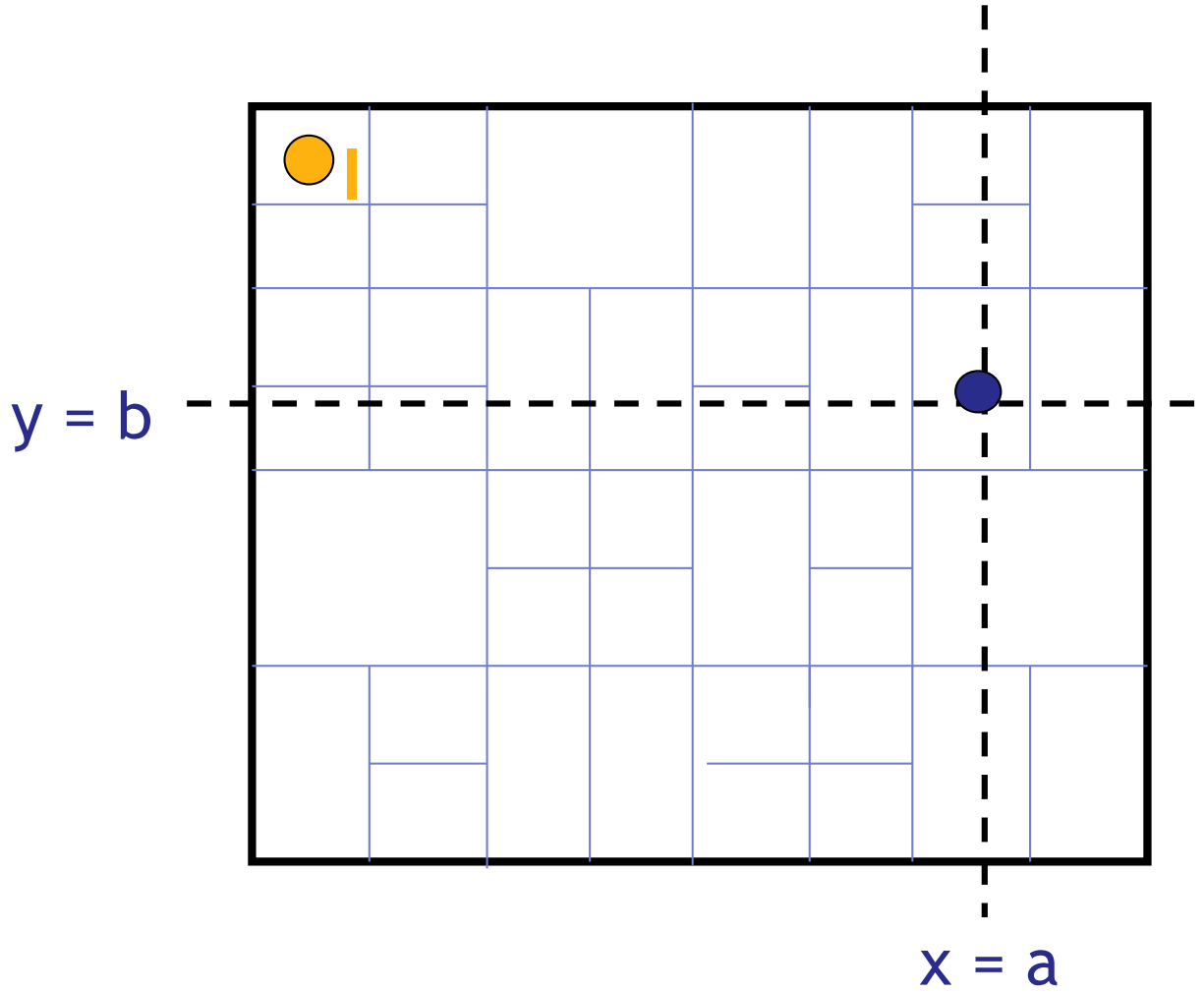


# CAN: Storing Values

node I::insert(K,V)

a = $h_x$(K)

b = $h_y$(K)

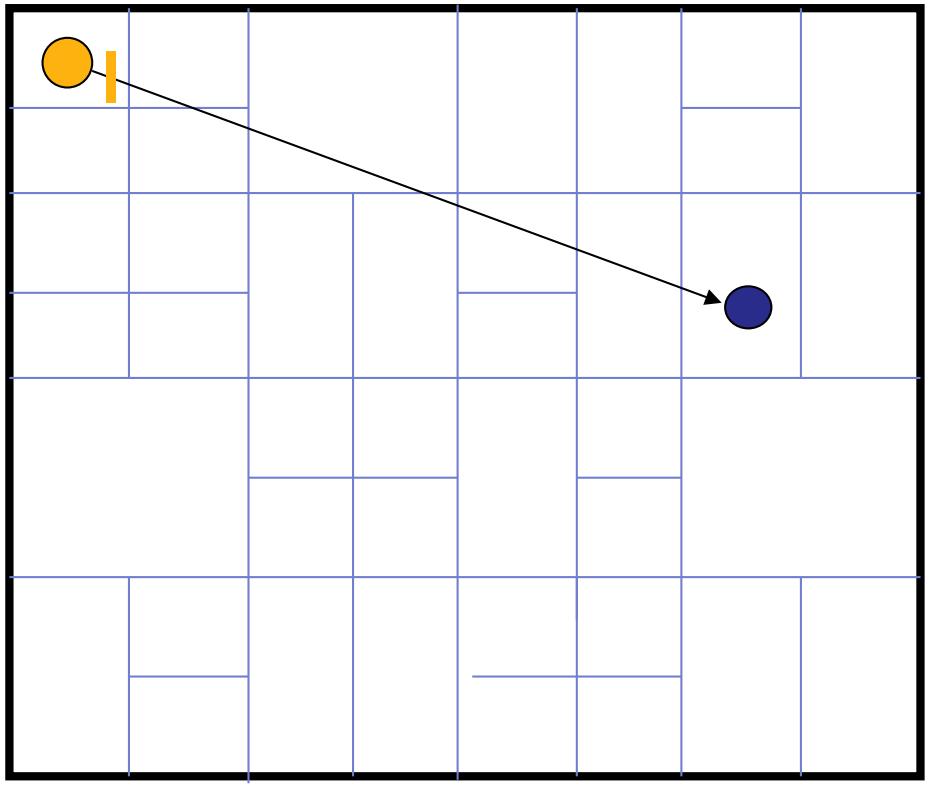


# CAN: Storing Values

node I::insert(K,V)

(1) a = $h_x(K)$
b = $h_y(K)$

(2) route(K,V) -> (a,b)



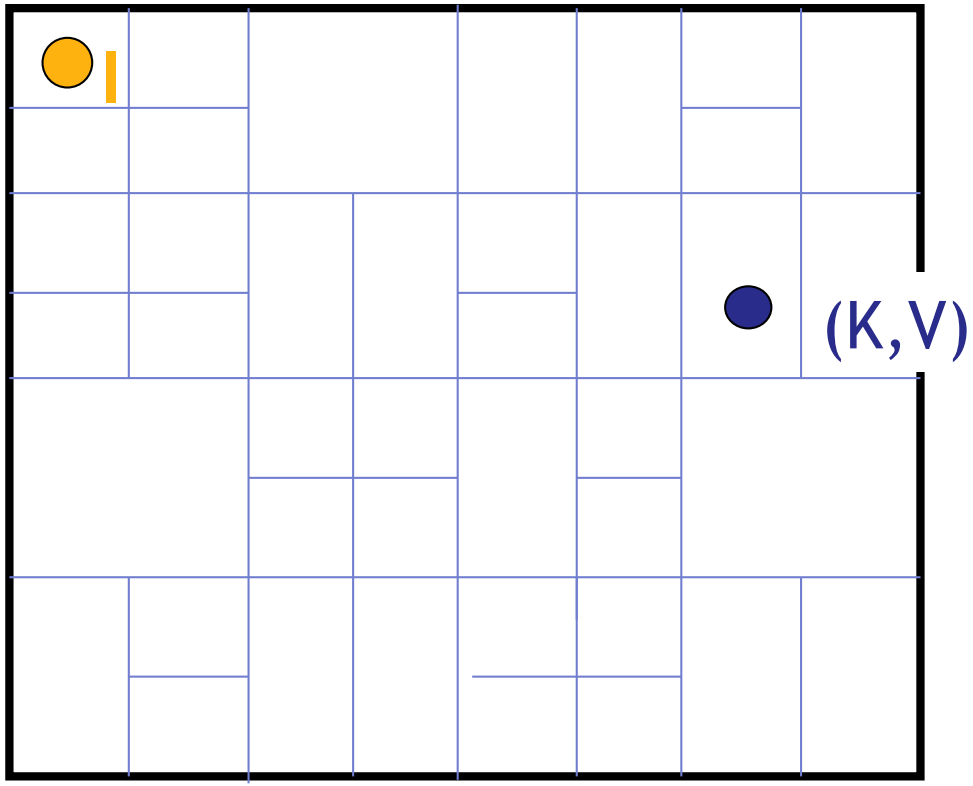
# CAN: Storing Values

node I::insert(K,V)

(1) a = $h_x(K)$
b = $h_y(K)$

(2) route(K,V) -> (a,b)

(3) (a,b) stores (K,V)

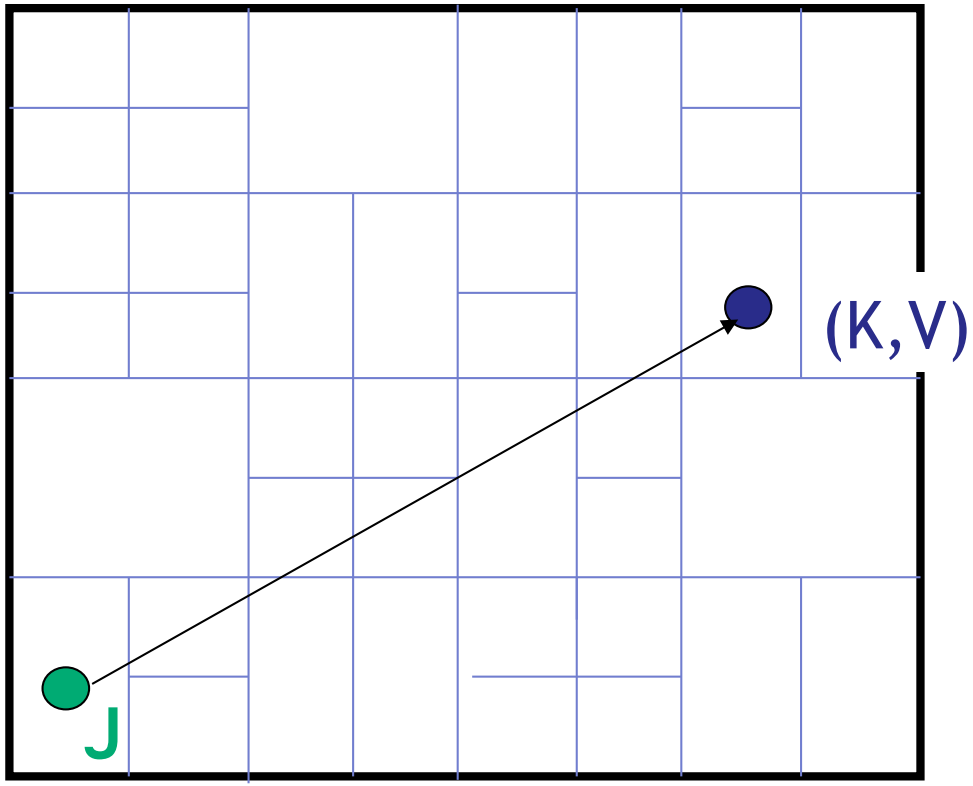


# CAN: Retrieving Values

node J::retrieve(K)

(1) a = $h_x(K)$
b = $h_y(K)$

(2) route "retrieve(K)" to (a,b)



# CAN: Improvements

- Possible to increase number of dimensions *d*
  - Small increase in routing table size
  - Shorter routing path, more neighbors for fault tolerance
- Multiple realities (= coordinate spaces)
  - Use more hash functions
  - Same properties as increased dimensions
- Routing weighted by round-trip times
  - Take into account network topology
  - Forward to the “best” neighbor

# CAN: More Improvements

- Use well-known landmark servers (e.g., DNS roots)
  - Nodes join CAN in different areas, depending on distance to landmarks
    - Pick points “near” landmark
  - Idea: Geographically close nodes see same landmarks
- Uniform partitioning
  - New node splits the largest zone in the neighborhood instead of the zone of the responsible node

# CAN: Performance

- State information at node *O(d)*
  - Number of dimensions is *d*
  - Need two neighbors in all coordinate axis
  - Independent of the number of nodes!

- Routing takes $O(dn^{1/d})$ hops
  - Network has *n* nodes
  - Multiple dimensions and realities improve this
  - For routing: multiple dimensions are better
  - But: multiple realities improve availability and fault tolerance

# Tapestry

- Tapestry developed at UC Berkeley(!)
  - Different group from CAN developers
- Tapestry developed in 2000, but published in 2004
  - Originally only as technical report, 2004 as journal article
- Many follow-up projects on Tapestry
  - Example: OceanStore

- Tapestry based on work by Plaxton et al.
- Plaxton network has also been used by Pastry
- Pastry was developed at Microsoft Research and Rice University
  - Difference between Pastry and Tapestry minimal
  - Tapestry and Pastry add dynamics and fault tolerance to Plaxton network

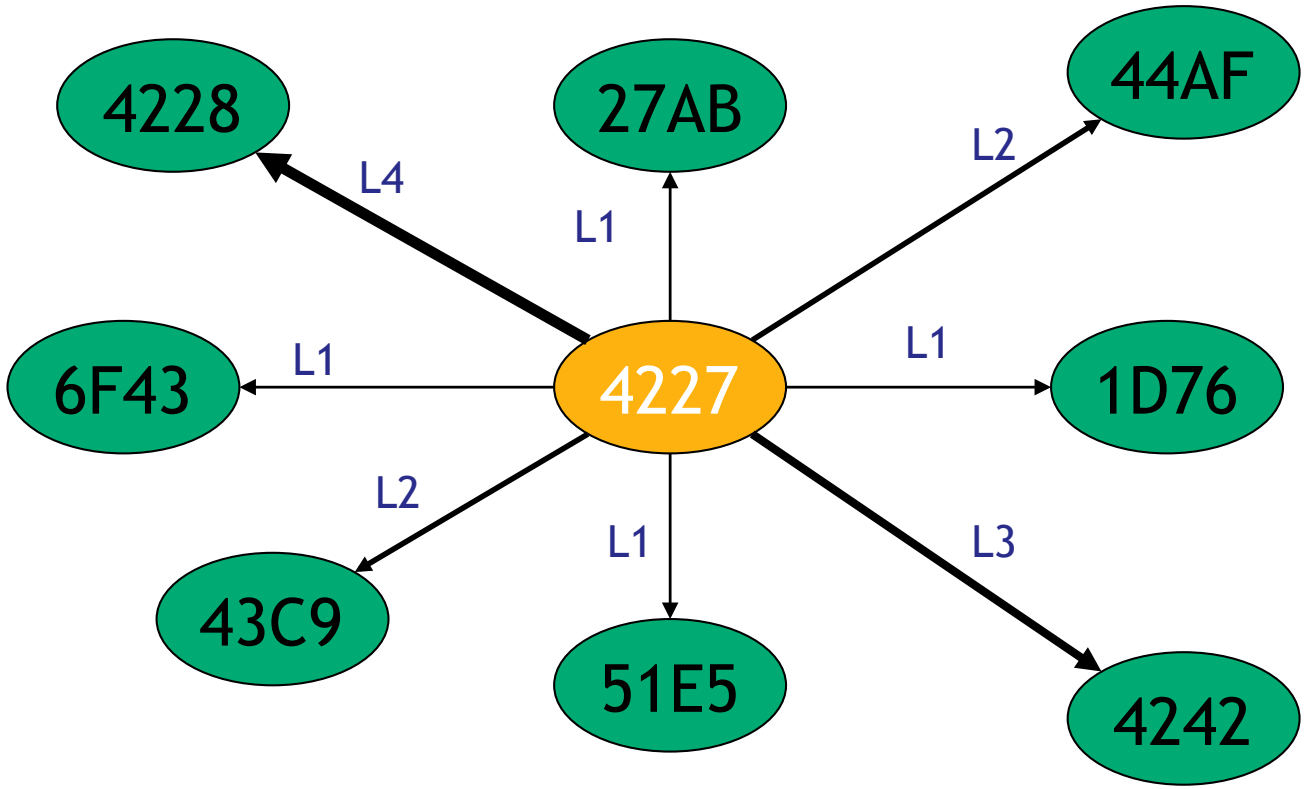
# Tapestry: Plaxton Network

- Plaxton network (or Plaxton mesh) based on prefix routing (similar to IP address allocation)
  - Prefix and postfix are functionally identical
  - Tapestry originally postfix, now prefix?!?
- Node ID and object ID hashed with SHA-1
  - Expressed as hexadecimal (base 16) numbers (40 digits)
  - Base is very important, here we use base 16

- Each node has a neighbor map with multiple levels
  - Each level represents a matching prefix up to digit position in ID
  - A given level has number of entries equal to the base of ID
  - $i^{th}$ entry in $j^{th}$ level is closest node which starts *prefix(N,j-1)+"i"*
  - Example: 9th entry of 4th level for node 325AE is the closest node with ID beginning with 3259

# Tapestry: Routing Mesh

- (Partial) routing mesh for a single node 4227
- Neighbors on higher levels match more digits

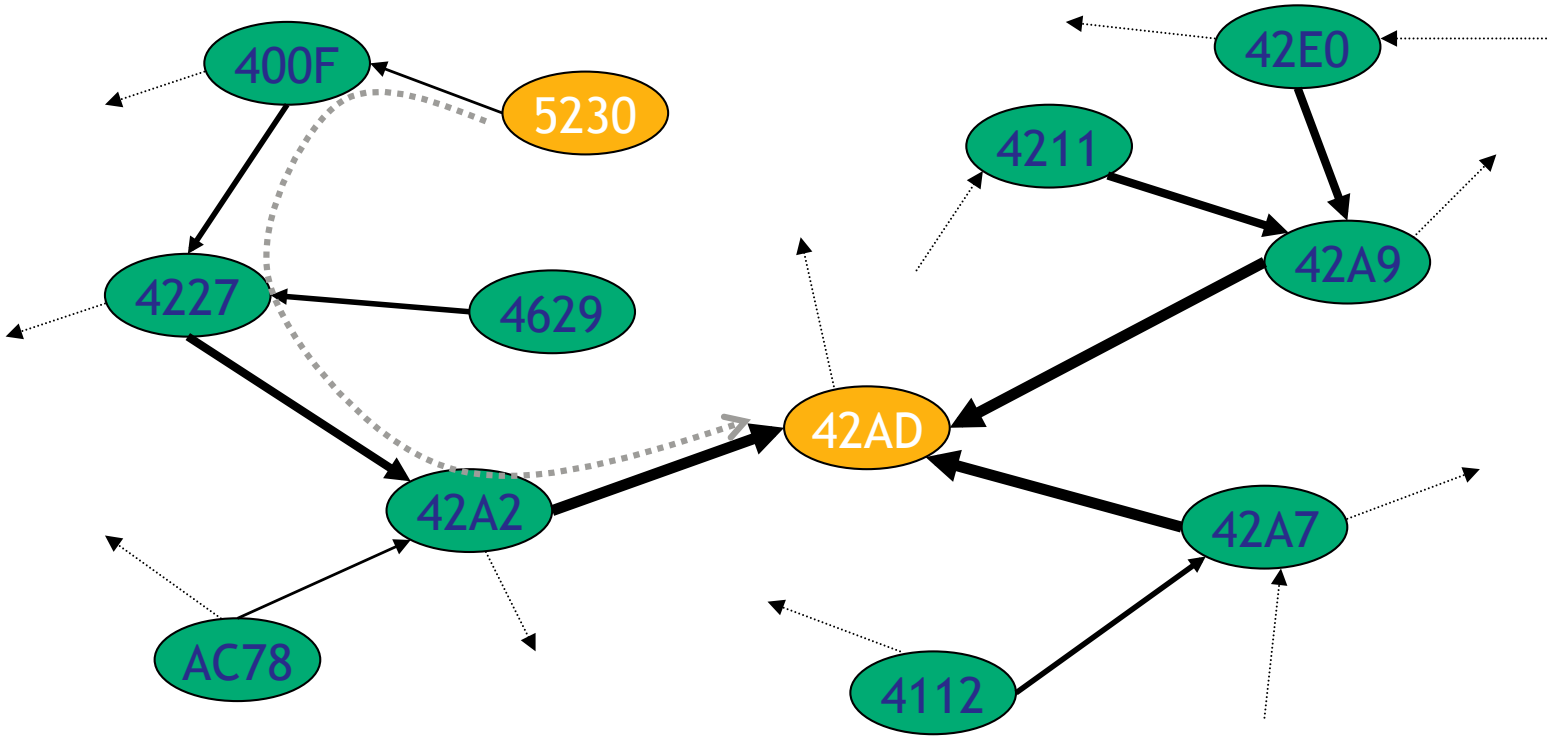


# Tapestry: Neighbor Map for 4227

| Level | 1 | 2 | 3 | 4 | 5 | 6 | 8 | A |
|---|---|---|---|---|---|---|---|---|
| 1 | 1D76 | 27AB | | | 51E5 | 6F43 | | |
| 2 | | | 43C9 | 44AF | | | | |
| 3 | | | | | | | | 42A2 |
| 4 | | | | | | | 4228 | |

- There are actually 16 columns in the map (base 16)
- Normally more (most?) entries would be filled
- Tapestry has neighbor maps of size 40 x 16

# Tapestry: Routing Example

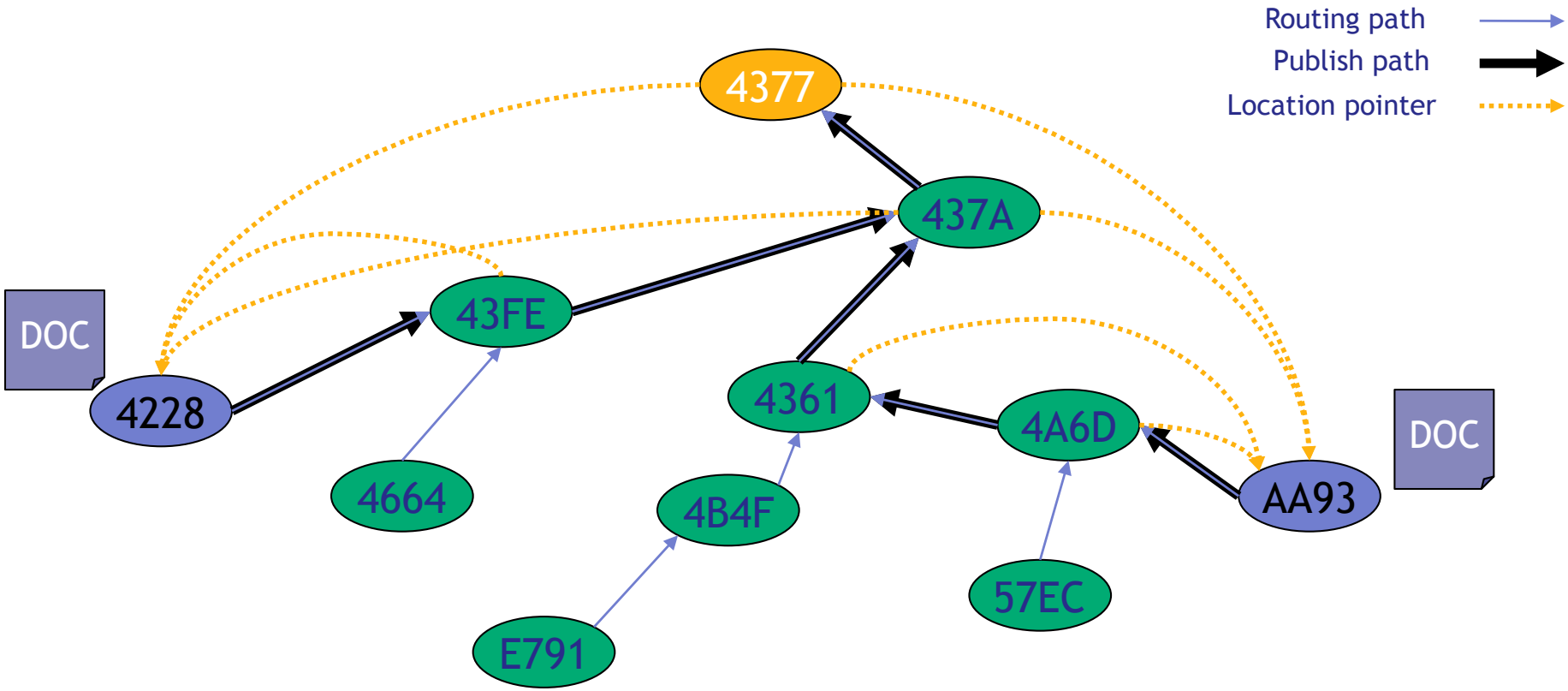



- Route message from 5230 to 42AD
- Always route to node closer to target
  - At $n^{th}$ hop, look at $n+1^{th}$ level in neighbor map --> “always” one digit more
- Not all nodes and links are shown

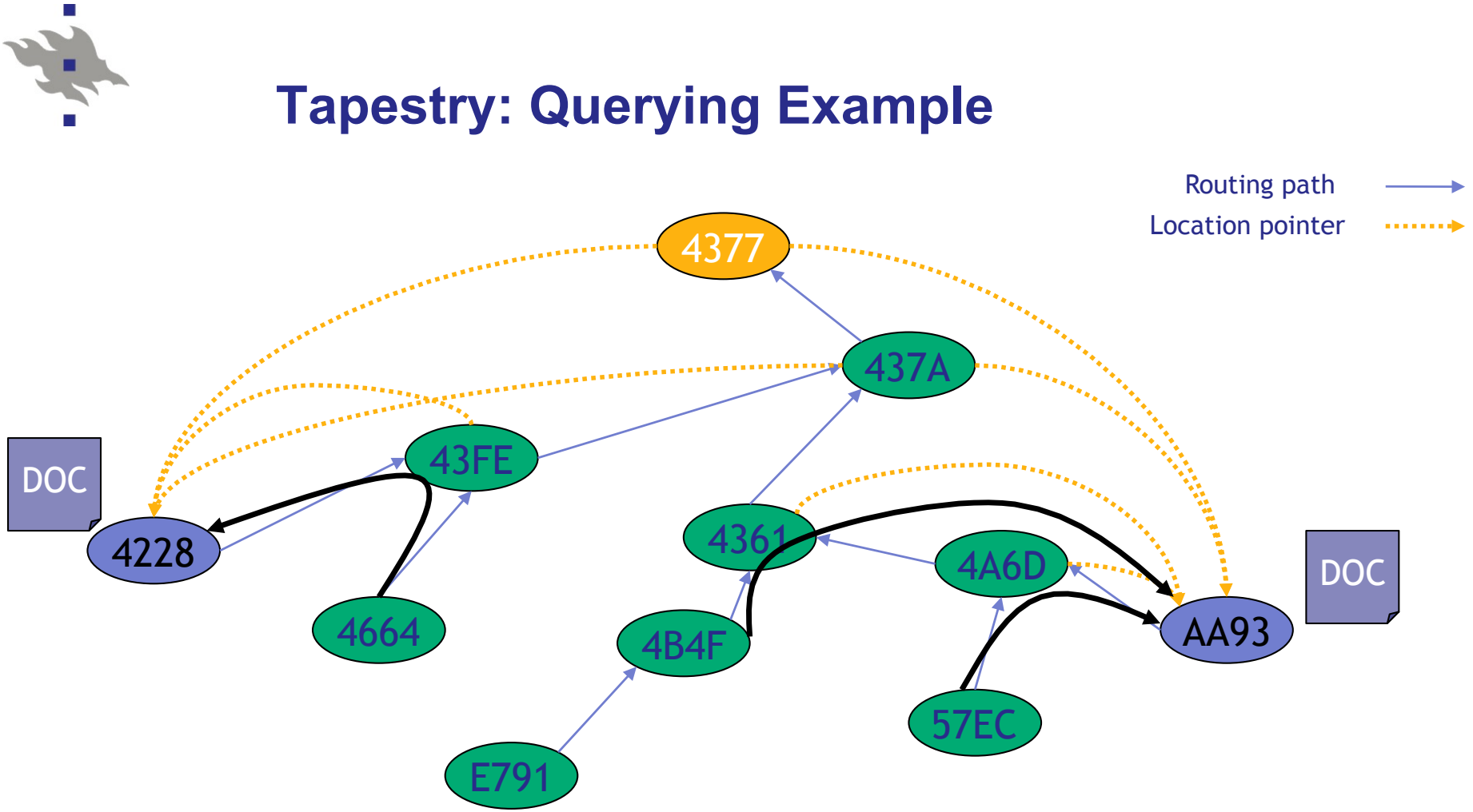
# Tapestry: Properties

- Node responsible for objects  which have same ID
  - Unlikely to find such node for every object
  - Node responsible also for “nearby” objects (surrogate routing, see below)
- Object publishing:
  - Responsible nodes store only pointers
    - Multiple copies of object possible
    - Each copy must publish itself
  - Pointers cached along the publish path
  - Queries routed towards responsible node
  - Queries “often” hit cached pointers
    - Queries for same object go (soon) to same nodes
- Note: Tapestry focuses on storing objects
  - Chord and CAN focus on values, but in practice no difference

# Tapestry: Publishing Example




- Two copies of object “DOC” with ID 4377 created at AA93 and 4228
- AA93 and 4228 publish object DOC, messages routed to 4377
  - Publish messages create location pointers on the way
- Any subsequent query can use location pointers



# Tapestry: Querying Example



- Requests initially route towards 4377
- When they encounter the publish path, use location pointers to find object
- Often, no need to go to responsible node
- Downside: Must keep location pointers up-to-date

# Tapestry: Making It Work

- Previous examples show a Plaxton network
  - Requires global knowledge at creation time
  - No fault tolerance, no dynamics
- Tapestry adds fault tolerance and dynamics
  - Nodes join and leave the network
  - Nodes may crash
  - Global knowledge is impossible to achieve
- Tapestry picks closest nodes for neighbor table
  - Closest in IP network sense (= shortest RTT)
  - Network distance (usually) transitive
    - If A is close to B, then B is also close to A
  - Idea: Gives best performance

# Tapestry: Fault-Tolerant Routing

- Tapestry keeps mesh connected with keep-alives
  - Both TCP timeouts and UDP “hello” messages
  - Requires extra state information at each node
- Neighbor table has backup neighbors
  - For each entry, Tapestry keeps 2 backup neighbors
  - If primary fails, use secondary
    - Works well for uncorrelated failures
- When node notices a failed node, it marks it as invalid
  - Most link/connection failures short-lived
  - Second chance period (e.g., day) during which failed node can come back and old route is valid again
  - If node does not come back, one backup neighbor is promoted and a new backup is chosen

# Tapestry: Fault-Tolerant Location

- Responsible node is a single point of failure
- Solution: Assign multiple roots per object
  - Add *"salt"* to object name and hash as usual
  - Salt = globally constant sequence of values (e.g., 1, 2, 3, …)
- Same idea as CAN's multiple realities
- This process makes data more available, even if the network is partitioned
  - With $s$ roots, availability is $P \approx 1 - (1/2)^s$
  - Depends on partition
- These two mechanisms "guarantee" fault-tolerance
  - In most cases :-)
  - Problem: If the only out-going link fails…

# Tapestry: Surrogate Routing

- Responsible node is node with same ID as object
  - Such a node is unlikely to exist
- Solution: surrogate routing
- What happens when there is no matching entry in neighbor map for forwarding a message?
- Node picks (deterministically) one entry in neighbor map
  - Details are not explained in the paper :(
- Idea: If “missing links” are deterministically picked, any message for that ID will end up at same node
  - This node is the surrogate
- If new nodes join, surrogate may change
  - New node is neighbor of surrogate

# Tapestry: Performance

- Messages routed in $O(log_b\ N)$ hops
  - At each step, we resolve one more digit in ID
  - $N$ is the size of the namespace (e.g, SHA-1 = 40 digits)
  - Surrogate routing adds a bit to this, but not significantly
- State required at a node is $O(b\ log_b\ N)$
  - Tapestry has $c$ backup links per neighbor, $O(cb\ log_b\ N)$
  - Additionally, same number of backpointers

# DHT: Comparison

| | Chord | CAN | Tapestry |
|---|---|---|---|
| Type of network | Ring | N-dimensional | Prefix routing |
| Routing | $O(\log n)$ | $O(d \cdot n^{1/d})$ | $O(\log_b N)$ |
| State | $O(\log n)$ | $O(d)$ | $O(b \cdot \log_b N)$ |
| Caching efficient | + | ++ | ++ |
| Robustness | -/+ | +++ | ++ |
| IP Topology-Aware | N | N/Y | Y |
| Used for other projects | +++ | -- | ++ |

Note: *n* is number of nodes, *N* is size of Tapestry's namespace

# Other DHTs

- Many other DHTs exist too
  - Pastry, similar to Tapestry
  - Kademlia, uses XOR metric
  - Kelips, group nodes into *k* groups, similar to KaZaA
  - Plus some others…
- Overnet P2P network (also eDonkey) uses Kademlia
  - Wide-spread deployed DHT
- All DHTs provide same API
  - In principle, DHT-layer is interchangeable

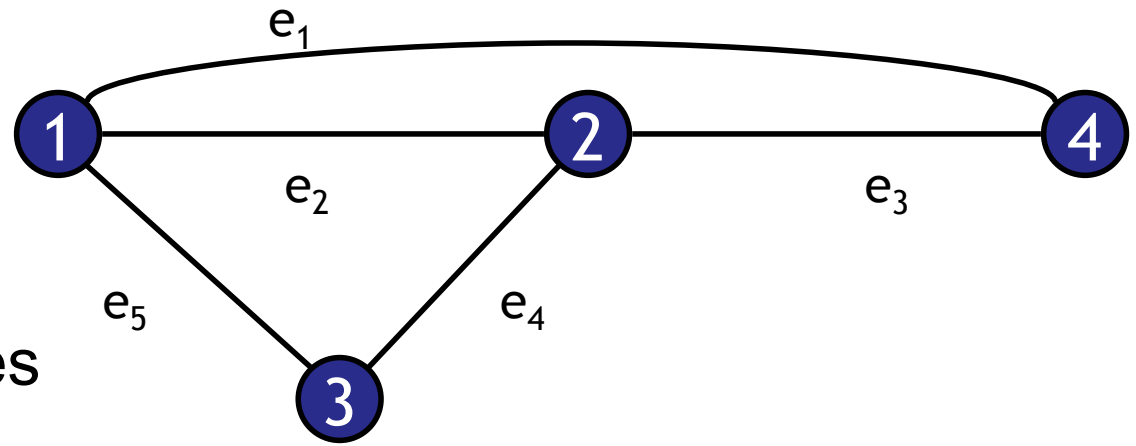
# Networks and Graphs

- Refresher of graph theory
- Graph families and models
  - Random graphs
  - Small world graphs
  - Scale-free graphs
- Graph theory and P2P
  - How are the graph properties reflected in real systems?

# What Is a Graph?

- Definition of a graph:
  Graph $G = (V, E)$ consists of two finite sets, set $V$ of vertices (nodes) and set $E$ of edges (arcs) for which the following applies:
  1. If $e \in E$, then exists $(v, u) \in V \times V$, such that $v \in e$ and $u \in e$
  2. If $e \in E$ and above $(v, u)$ exists, and further for $(x, y) \in V \times V$ applies $x \in e$ and $y \in e$, then $\{v, u\} = \{x, y\}$

Example graph with
4 vertices and 5 edges



# Properties of Graphs

- An edge $e \in E$ is directed if the start and end vertices in condition 2 above are identical: $v = x$ and $y = u$
- An edge $e \in E$ is undirected if $v = x$ and $y = u$ as well as $v = y$ and $u = x$ are possible
- A graph $G$ is directed (undirected) if the above property holds for all edges
- A *loop* is an edge with identical endpoints
- Graph $G_1 = (V_1, E_1)$ is a subgraph of $G = (V, E)$, if $V_1 \subseteq V$ and $E_1 \subseteq E$ (such that conditions 1 and 2 are met)

# Important Types of Graphs

- Vertices $v, u \in V$ are connected if there is a path from $v$ to $u$: $(v, v_2), (v_2, v_3), \ldots, (v_{k-1}, u) \in E$
- Graph $G$ is connected if all $v, u \in V$ are connected
- Undirected, connected, acyclic graph is called a tree
  - Sidenote: Undirected, acyclic graph which is not connected is called a forest
- Directed, connected, acyclic graph is also called DAG
  - DAG = directed, acyclic graph (connected is “assumed”)
- An induced graph $G(V_C) = (V_C, E_C)$ is a graph $V_C \subseteq V$ and with edges $E_C = \{e = (i, j) \mid i, j \in V_C\}$
- An induced graph is a component if it is connected

# Vertex Degree

- In graph $G = (V, E)$, the degree of vertex $v \in V$ is the total number of edges $(v, u) \in E$ and $(u, v) \in E$
  - Degree is the number of edges which touch a vertex
- For directed graph, we distinguish between in-degree and out-degree
  - In-degree is number of edges coming to a vertex
  - Out-degree is number of edges going away from a vertex
- The degree of a vertex can be obtained as:
  - Sum of the elements in its row in the incidence matrix
  - Length of its vertex incidence list

# Important Graph Metrics

- Distance: *d(v, u)* between vertices *v* and *u* is the length of the shortest path between *v* and *u*
- Average path length: Sum of the distances over all pairs of nodes divided by the number of pairs
- Diameter: *d(G)* of graph *G* is the maximum of *d(v, u)* for all $v, u \in V$

# Six Degrees of Separation

- Famous experiment from 1960's (S. Milgram)
- Send a letter to random people in Kansas and Nebraska and ask people to forward letter to a person in Boston
  - Person identified by name, profession, and city
- Rule: Give letter only to people you know by first name and ask them to pass it on according to same rule
- Some letters reached their goal
- Letter needed six steps on average to reach the person
- Graph theoretically: Social networks have dense local structure, but (apparently) small diameter
- How to model such networks?

# Random Graphs

- Random graphs are first widely studied graph family
  - Many P2P networks choose neighbors more or less randomly
- Two different notations generally used:
  - Erdös and Renyi
  - Gilbert (we will use this)
- Gilbert's definition: Graph $G_{n,p}$ (with $n$ nodes) is a graph where the probability of an edge $e = (v, w)$ is $p$

Construction algorithm:

- For each possible edge, draw a random number
- If the number is smaller than $p$, then the edge exists
- $p$ can be function of $n$ or constant

# Basic Results for Random Graphs

Giant Connected Component:

*Let c > 0 be a constant and p = c/n. If c < 1 every component of $G_{n,p}$ has order O(log N) with high probability. If c > 1 then there will be one component of size n*(f(c) + O(1)) where f(c) > 0, with high probability. All other components have size O(log N)*

- In plain English: Giant connected component emerges with high probability when average degree is about 1

Node degree distribution

- If we take random node, how high is probability *P(k)* that node has degree *k*?
- Node degree is Poisson distributed $P(k) = \frac{c^k e^{-c}}{k!}$

# More Basic Results

## Diameter of a random graph

- If $pn/\log(n) \to \infty$ and $\log(n)/\log(pn) \to \infty$ then the diameter of $G_{n,p}$ is asymptotic to $\log(n)/\log(pn)$ with high probability

## Clustering coefficient

- Clustering coefficient measures number of edges between neighbors divided by maximum number of edges between them (clique-like)
- Clustering coefficient C(i) is defined as $C(i) = \frac{E(N(i))}{d(i)(d(i)-1)}$
  - $E(N(i))$ = number of edges between neighbors of $i$
  - $d(i)$ = degree of $i$
- Clustering coefficient of a random graph is asymptotically equal to $p$ with high probability

# Random Graphs: Summary

- Before random graphs, regular graphs were popular
  - Regular: Every node has same degree
- Random graphs have two advantages over regular graphs

1. Many interesting properties analytically solvable
2. Much better for applications, e.g., social networks

- Note: Does not mean social networks are random graphs; just that the properties of social networks are well-described by random graphs
- Question: How to model networks with local clusters and small diameter?
- Answer: Small-world networks

# Small-World Networks

- Developed/discovered by Watts and Strogatz (1998)
  - Over 30 years after Milgram's experiment!
- Watts and Strogatz looked at three networks
  - Film collaboration between actors
  - US power grid
  - Neural network of worm *C. elegans*
- Results:
  - Compared to a random graph with same number of nodes
  - Diameters similar, slightly higher for real graph
  - Clustering coefficient orders of magnitude higher

Definition of small-worlds network:

- Dense local clustering structure and small diameter comparable to that of a same-sized random graph

# Constructing Small-World Graphs

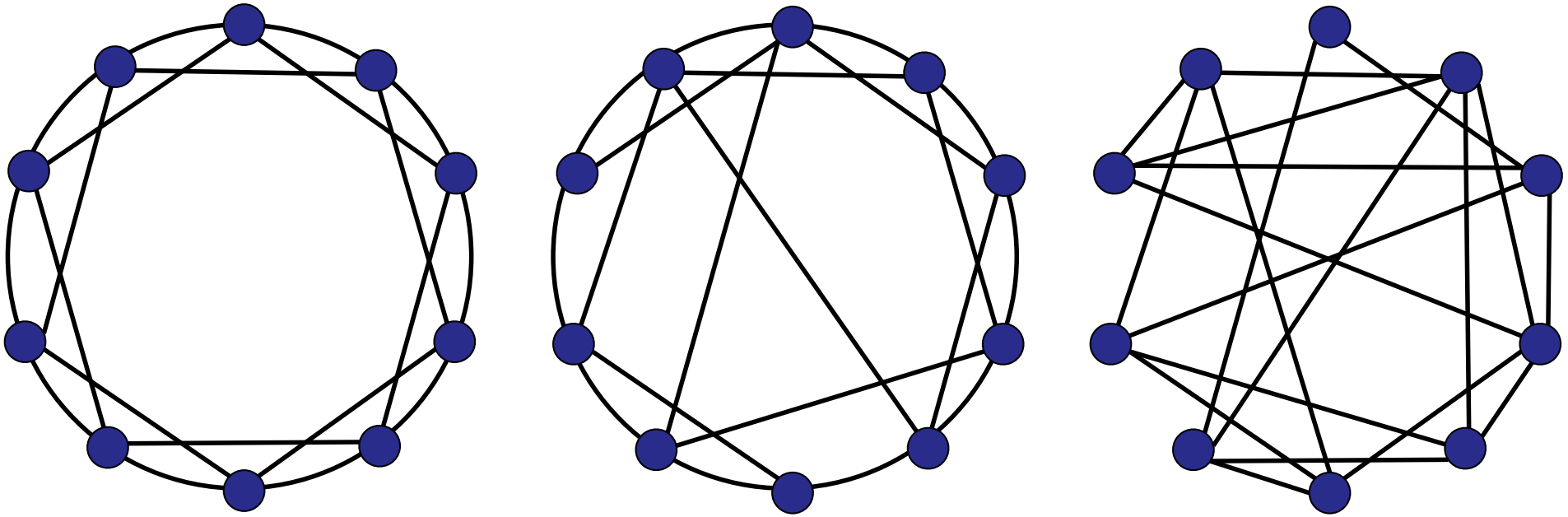
- Put all *n* nodes on a ring, number them consecutively from 1 to *n*
- Connect each node with its *k* clockwise neighbors
- Traverse around ring in clockwise order
- For every edge:
  - Draw random number *r*
  - If $r < p$, then re-wire edge by selecting a random target node from the set of all nodes (no duplicates)
  - Otherwise keep old edge
- Different values of *p* give different graphs
  - If *p* is close to 0, then original structure mostly preserved
  - If *p* is close to 1, then new graph is random
  - Interesting things happen when *p* is somewhere in-between

# Regular, Small-World, Random

Regular

Small-World

Random



p = 0

p = 1

# Problems with Small-World Graphs

Small-world graphs explain why:

- Highly clustered graphs can have short average path lengths

Small-world graphs do *NOT* explain why:

- This property emerges in real networks
  - Real networks are practically never ring-like

Further problem with small-world graphs:

- Nearly all nodes have same degree
- Not true for random graphs (*k* edges ~ $c^k/k!$)
- Is same true for real networks too?
- Let's look at the Internet…

# Internet

- Famous study by Faloutsos et al. (3 brothers! ;-) in 1999
- They examined Internet topology during 1998
  - AS-level topology, during 1998 Internet grew 45%

Motivation for work:

- What does the Internet look like?
- Are there any topological properties that don't change over time?
- How can I generate Internet-like graphs for simulations?

# Faloutsos Results

- 4 key properties, each follows a power-law
- Sort nodes according to their (out)degree

1. *Outdegree of a node is proportional to its rank to the power of a constant*
2. *Number of nodes with same outdegree is proportional to the outdegree to the power of a constant*
3. *Eigenvalues of a graph are proportional to the order to the power of a constant*
4. *Total number of pairs of nodes within a distance d is proportional to d to the power of a constant*

- Why would Internet obey such laws?

# Answer: Power-Law Networks

- Also known as scale-free networks
- Barabasi-Albert-Model

1. Network grows in time
2. New node has preferences to whom it wants to connect

- Preferential connectivity modeled as
  - Each new node wants to connect to *m* other nodes
  - Probability that an existing node *j* gets one of the *m* connections is proportional to its degree *d(j)*
- New nodes tend to connect to well-connected nodes
- Another way to express this is “rich get richer”

# Applications to Peer-to-Peer

- Small-world model explains why short paths exist
- Why can we find these paths?
  - Each node has only local information
  - Milgram's results showed first steps were the largest
- How to model this?
- Kleinberg's Small-World Model
  - Set of points in an *n* x *n* grid
  - Distance is the number of "steps" separating points
    - $d(i, j) = |x_i - x_j| + |y_i - y_j|$
- Construct graph as follows:
  - Every node *i* is connected to node *j* within distance *q*
  - For every node *i*, additional *q* edges are added. Probability that node *j* is selected is proportional to $d(i, j)^{-r}$, for some constant *r*

# Navigation in Kleinberg's Model

- We want to send a message to another node
- Algorithm is decentralized if sending node only knows:
  - Its local neighbors
  - Position of the target node on the grid
  - Locations and long-range contacts of all nodes who come in contact of the message (not needed below, actually)
- Can be shown: Number of messages needed is proportional to *O(log n)* (only one correct *r* per case)
- Practical algorithm: Forward message to contact who is closest to target
- Note: Kleinberg's model assumes some way of associating nodes with points in grid
  - Compare with CAN DHT in Chapter 3

# Power Law Networks and P2P

- Robustness comparison of random and power-law graphs
- Take network of 10000 nodes (random and power-law) and remove nodes randomly
- Random graph:
  - Take out 5% of nodes: Biggest component 9000 nodes
  - Take out 18% of nodes: No biggest component, all components between 1 and 100 nodes
  - Take out 45% of nodes: Only groups of 1 or 2 survive
- Power-law graph:
  - Take out 5% of nodes: Only isolated nodes break off
  - Take out 18% of nodes: Biggest component 8000 nodes
  - Take out 45% of nodes: Large cluster persists, fragments small
- Recall Gnutella: *Applies ONLY for random failures*

# Summary of Graphs

- Three kinds of graph models:
  - Random graph
  - Small-World
  - Power-Law (Scale-Free)
- Small-world graphs explain why we can have high clustering and short average paths
- Power-law graphs explain how graphs are built in many real networks

# Chapter Summary

- Searching and addressing
  - Fundamental difference
  - Unstructured vs. structured networks
- Distributed Hash Tables
  - DHT provides a key to value mapping
  - Three examples: Chord, CAN, Tapestry
- Different networks and graphs
  - Random, small world, scale-free networks