1. (a) Give a formal definition for the following finite automaton. What is the sequence of states the automaton enters on the following inputs: 0101, 1010 and 000111. What is the language recognised by the automaton?

(b) Let \( M = (\{ q_0, q_1, q_2 \}, \{ 0, 1 \}, \delta, q_0, \{ q_1 \}) \), where \( \delta \) is as follows:

\[
\begin{array}{c|cc}
& 0 & 1 \\
\hline
q_0 & q_1 & q_0 \\
q_1 & q_2 & q_1 \\
q_2 & q_1 & q_2
\end{array}
\]

Draw the automaton as a state diagram. What is the sequence of states the automaton enters on the following inputs: 01010, 1010 and 000111. What is the language recognised by the automaton?

2. For each of the following languages over the alphabet \{ a, b, c \}, give a finite automaton recognising the language (as a state diagram):
   (a) strings that end with “abc”
   (b) strings that begin with “abc”
   (c) strings where each odd-numbered position contains character b.

3. Suppose a finite automaton \( M \) is given.
   (a) How can you easily decide whether \( \varepsilon \in L(M) \)?
   (b) Give an algorithm that decides whether \( L(M) = \emptyset \). How would you augment your algorithm so that in case \( L(M) \neq \emptyset \) it also returns some string belonging to \( L(M) \)?

4. Prove that the language \{ 0^n1^n \mid n \in \mathbb{N} \} is not regular.
   \textit{Hint:} Extend the transition function by defining \( \delta^*(q, w) \), for any \( q \in Q \) and \( w \in \Sigma^* \), to be the state where the automaton would end up if it started in state \( q \) and received as input the string \( w \). Suppose some \( n \neq m \) satisfy \( \delta^*(q_0, 0^n) = \delta^*(q_0, 0^m) = q \). Is the state \( \delta^*(q, 1^n) \) an accept state or not?