Maximum from this exam is 24 points.

1. [8 points] For each of the following languages over the alphabet \{a, b, c\}, give a context-free grammar that generates it.
   
   (a) \( A_1 = \{ a^n b^n \mid n \geq 0 \} \)
   
   (b) the language \( A_2 \) consist of all the strings whose length is an integer multiple of three
   
   (c) \( A_3 = \{ a^m b^m c^n \mid m, n \geq 0 \} \)
   
   (d) the language \( A_4 \) consist of strings in which the number of ‘a’ characters is same as the numbers of ‘b’ and ‘c’ characters put together.

2. [6 points] Let the language \( B \) over the alphabet \{a, b, c\} consist of all the strings in which the number of ‘a’ characters is at least as large as the numbers of ‘b’ and ‘c’ characters put together. Prove that the language \( B \) is not regular.
   
   In your proof you may use any known general properties of regular languages, but not results that directly say that a given language is not regular.

3. [4 points] Give a push-down automaton that recognises the language \( B \) defined in Problem 2. Explain with a couple of sentences how your automaton works.

4. [6 points] **Halting problem.** Define the Turing machine halting problem precisely as a formal language (but do not worry about the details of encodings). Explain the content of the definition in plain English. Is this formal language decidable? Is it Turing-recognisable? (You do not need to justify your answers to these questions.) Explain briefly the meaning of these results for normal programming languages.

(Tehtävät suomeksi kääntöpuolella)