1. Let the language $A$ over alphabet $\{0, 1\}$ consist of strings that end in zero. Further, let the language $B$ consist of strings that end in 01. The languages $A$ and $B$ can be recognised with DFAs $M_A$ and $M_B$, respectively:

DFA $M_A$.

DFA $M_B$.

Construct an NFA for the language $A \circ B$ by applying the construction given in Sipser, pp. 60–61. Show the computation tree of the NFA for input strings 001001 and 1110011011001.

2. Give a regular expression for each of the following languages over the alphabet $\Sigma = \{0, 1\}$:

(a) strings that contain 000 or 111 as substring
(b) strings that contain both 000 and 111 as substring
(c) strings of length at least two where the last two characters are the same (and in same order) as the first two
(d) strings that do not contain 000 as substring.

3. (Sipser Problem 1.31) Define the reverse of a language $A$ as

$$A^R = \{ w^R \mid w \in A \}.$$ 

Show that if $A$ is regular, then so is $A^R$.

4. (a) We say that a string $w$ is a prefix of a string $x$, if a string $z$ exists such that $x = wz$. For any language $A$ over alphabet $\Sigma$, we define its set of prefixes as

$$\text{PREFIX}(A) = \{ w \in \Sigma^+ \mid \text{there exists } z \in \Sigma^* \text{ such that } wz \in A \}.$$ 

Prove that if $A$ is regular, then so is $\text{PREFIX}(A)$.

(b) We say that a string $w$ is a suffix of a string $x$, if a string $z$ exists such that $x = zw$. For any language $A$, we define its set of suffixes as

$$\text{SUFFIX}(A) = \{ w \in \Sigma^+ \mid \text{there exists } z \in \Sigma^* \text{ such that } zw \in A \}.$$ 

Prove that if $A$ is regular, then so is $\text{SUFFIX}(A)$. Hint: you may apply part (a) together with the result from Problem 3.