Basic exercises

Solve these by yourself. If there is anything unclear you can ask about it during the exercise session.

1. Create a DFA for the language \((0 \cup 01)^*\) in alphabet \(\{0, 1\}\). Try to make the DFA as simple as possible. Then construct an NFA for the same language, using the construction for converting a regular expression into an NFA (proof of Lemma 1.55 in Sipser’s book).

2. Convert your NFA from Problem 1 into a DFA using the procedure from Sipser’s Theorem 1.39. Compare the result with the DFA you obtained directly in Problem 1.

3. Convert the following DFA into a regular expression using the method given in Lemma 1.60 of Sipser’s book:

   ![DFA Diagram]

Discussion problems

Read the following problems and make sure you are familiar with the necessary basic concepts. You are not expected to solve the problems by yourself; we shall discuss them together.

4. (Sipser Problem 1.55) A natural number \(p\) is a pumping length for a language \(A\) if any string \(w \in A\) with \(|w| \geq p\) can be pumped (as in the statement of the Pumping lemma). The minimum pumping length for \(A\) is the smallest \(p\) such that \(p\) is a pumping length for \(A\). For each of the following languages, determine its minimum pumping length and justify your answer.

   \[
   \begin{align*}
   C_1 &= (01)^* \\
   C_2 &= 1^*01^*01^* \\
   C_3 &= \varepsilon \\
   C_4 &= 00100.
   \end{align*}
   \]

5. Which of the following languages over the alphabet \(\Sigma = \{0, 1\}\) are regular?

   \[
   \begin{align*}
   A_1 &= \{a^n b^n c^n \mid n \in \mathbb{N}\} \\
   A_2 &= \{0^n10^n \mid n \in \mathbb{N}\} \\
   A_3 &= \{0^n0^n \mid n \in \mathbb{N}\} \\
   A_4 &= \{ww^R \mid w \in \Sigma^*\} \\
   A_5 &= \{ww^R \mid w, u \in \Sigma^+\}.
   \end{align*}
   \]

   Justify your answers e.g. by giving a finite automaton or applying the pumping lemma.

6. Prove that the following languages over the alphabet \(\Sigma = \{0, 1\}\) are not regular:

   \[
   \begin{align*}
   B_1 &= \{0^m1^k0^n \mid m, k, n \in \mathbb{N} \text{ and } m \neq n\} \\
   B_2 &= \{w \in \Sigma^* \mid w \neq w^R\}.
   \end{align*}
   \]

   You may use the results from the previous problem and the closure properties of regular languages.