For ease of grading, please write your answer to each of the problems 1, 2 and 3 on its own sheet of paper. When asked to show an automaton, give a state diagram (ie, “picture”) rather than a formal description. Make sure you answer all parts of all the problems. The maximum grade is 24 points.

1. [4+4 points] For the following two languages over the alphabet \{ a, b \}, give both a deterministic finite automaton and a regular expression.
   (a) strings that contain the substring “aabb”
   (b) strings in which the number of character ‘a’ is evenly divisible by three.

2. [3+3 points]
   (a) By using the general method given in the textbook, give a nondeterministic finite automaton for the language \( 0^* (1 \cup 00)^* \). You do not need to show intermediate steps, assuming the final result can be clearly seen as a result of the proper procedure.
   (b) By using the general method given in the textbook, transform the nondeterministic finite automaton below into a deterministic one. You do not need to show intermediate steps, assuming the final result can be clearly seen as a result of the proper procedure. What is the language recognised by the automaton? (Describe it verbally or by a regular expression.)

3. [6+4 points]
   (a) Which of the following statements are true and which are false? For each statement, justify your answer by giving a proof or a counter-example. You may use any properties of regular languages that are proven in the textbook.
      i. If \( A \cup B \) is regular and \( A \) is regular, then \( B \) is regular.
      ii. If \( A \cup B \) is not regular and \( B \) is regular, then \( A \) is not regular.
      iii. If \( A \) is regular and \( B \) is not regular, then \( A \cup B \) is not regular.
   (b) The language \( A \) over the alphabet \{ a, b \} consists of all the strings in which there are more ‘a’ characters than ‘b’ characters. Prove that \( A \) is not a regular language. You may use the pumping lemma and any other known general properties of regular languages, but not results that directly state that a certain language is not regular.