Basic exercises

The first three problems are basic applications of the material from the text book. Solve them by yourself; if there is anything unclear you can ask about it during the exercise session.

1. Give a state diagram for the NFA \( N = (Q, \Sigma, \delta, q_0, F) \) where \( Q = \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7 \} \), \( \Sigma = \{ a, b, c \} \), \( F = \{ q_4 \} \) and \( \delta \) is as follows:

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( \varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_0 )</td>
<td>( \emptyset )</td>
<td>( \emptyset )</td>
<td>( \emptyset )</td>
<td>( { q_1, q_5 } )</td>
</tr>
<tr>
<td>( q_1 )</td>
<td>( { q_1, q_2 } )</td>
<td>( { q_1 } )</td>
<td>( { q_1 } )</td>
<td>( \emptyset )</td>
</tr>
<tr>
<td>( q_2 )</td>
<td>( \emptyset )</td>
<td>( { q_3 } )</td>
<td>( \emptyset )</td>
<td>( { q_3 } )</td>
</tr>
<tr>
<td>( q_3 )</td>
<td>( \emptyset )</td>
<td>( \emptyset )</td>
<td>( { q_4 } )</td>
<td>( \emptyset )</td>
</tr>
<tr>
<td>( q_4 )</td>
<td>( { q_4 } )</td>
<td>( { q_4 } )</td>
<td>( { q_4 } )</td>
<td>( \emptyset )</td>
</tr>
<tr>
<td>( q_5 )</td>
<td>( { q_5 } )</td>
<td>( { q_5 } )</td>
<td>( { q_5, q_6 } )</td>
<td>( \emptyset )</td>
</tr>
<tr>
<td>( q_6 )</td>
<td>( \emptyset )</td>
<td>( { q_7 } )</td>
<td>( \emptyset )</td>
<td>( { q_7 } )</td>
</tr>
<tr>
<td>( q_7 )</td>
<td>( { q_4 } )</td>
<td>( \emptyset )</td>
<td>( \emptyset )</td>
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</table>

What is the language recognized by the automaton?

2. Let the language \( A \) over alphabet \( \{ 0, 1 \} \) consist of string that end in zero. Further, let the language \( B \) consist of strings that end in 01. The languages \( A \) and \( B \) can be recognized with the following NFAs:

(a) Construct an NFA for the language \( A \circ B \) using the construction from lectures (ss. 82–83, Sipser ss. 60–61). Give also the formal description of the NFA.

(b) Show the computation tree (lectures s. 62, Sipser s. 49) for the input 001101.

(c) Construct a DFA for the same language using the construction from lectures (ss. 72–74, Sipser ss. 55–56).

3. (Sipser Exercise 1.15) In the construction for the NFA for \( A^* \) (ss. 83–84, Sipser ss. 62–63), the NFA for \( A \) was augmented with a new start state \( q_0 \) to make sure that the empty string \( \varepsilon \) is accepted. What is wrong with just making the original start state of \( N_1 \) an accept state? Give an example where this would give an incorrect result.

Continues on the next page!
Discussion problems

Read the following problems and make sure you are familiar with the necessary basic concepts. You are not expected to solve the problems by yourself; we shall discuss them together.

4. (Sipser Problem 1.38) A nondeterministic finite automaton accepts the input if at least one of the possible computations ends in an accept state. Let us consider a new kind of automaton, an \( \forall \)NFA, which is otherwise like a NFA but accepts only if all possible computations on the input end up in an accept state.

Show that a language can be recognized by an \( \forall \)NFA if and only if it is regular.

5. (Sipser Problem 1.31) Define the reverse of a language \( A \) as

\[
A^R = \{ w^R \mid w \in A \}.
\]

Show that if \( A \) is regular, then so is \( A^R \).

6. (a) We say that a sting \( w \) is a prefix of a string \( x \), if a string \( z \) exists such that \( x = wz \). For any language \( A \) over alphabet \( \Sigma \), we define its set of prefixes as

\[
\text{PREFIX}(A) = \{ w \in \Sigma^* \mid \text{there exists } z \in \Sigma^* \text{ such that } wz \in A \}.
\]

Prove that if \( A \) is regular, then so is \( \text{PREFIX}(A) \).

(b) We say that a sting \( w \) is a suffix of a string \( x \), if a string \( z \) exists such that \( x = zw \). For any language \( A \), we define its set of suffixes as

\[
\text{SUFFIX}(A) = \{ w \in \Sigma^* \mid \text{there exists } z \in \Sigma^* \text{ such that } zw \in A \}.
\]

Prove that if \( A \) is regular, then so is \( \text{SUFFIX}(A) \).

7. Let the language \( A_n \) over alphabet \{ a, b \} consist of strings where the \( n \)th symbol from the end is \( a \).

(a) Construct an NFA recognizing \( A_n \) with at most \( n + 1 \) states.

(b) Show that any DFA recognizing \( A_n \) must have at least \( 2^n \) states.