582206 Models of Computation (Autumn 2011)

Homework 4 (27–30 September)

1. Draw the state diagram of the NFA $N = (Q, \Sigma, \delta, q_0, F)$ where $Q = \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7 \}$, $\Sigma = \{ a, b, c \}$, $F = \{ q_4 \}$ and $\delta$ is as follows:

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>${ q_1, q_5 }$</td>
</tr>
<tr>
<td>$q_1$</td>
<td>${ q_1, q_2 }$</td>
<td>${ q_1 }$</td>
<td>${ q_1 }$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$q_2$</td>
<td>$\emptyset$</td>
<td>${ q_3 }$</td>
<td>$\emptyset$</td>
<td>${ q_3 }$</td>
</tr>
<tr>
<td>$q_3$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>${ q_4 }$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$q_4$</td>
<td>${ q_4 }$</td>
<td>${ q_4 }$</td>
<td>${ q_4 }$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$q_5$</td>
<td>${ q_5 }$</td>
<td>${ q_5 }$</td>
<td>${ q_5, q_6 }$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$q_6$</td>
<td>$\emptyset$</td>
<td>${ q_7 }$</td>
<td>$\emptyset$</td>
<td>${ q_7 }$</td>
</tr>
<tr>
<td>$q_7$</td>
<td>${ q_4 }$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
</tbody>
</table>

Show the computation tree of the NFA on input acbca. What is the language recognised by the NFA?

2. Transform the NFA given below into a DFA using the method given in the text book.

3. Which of the following claims are true? In each case, prove the claim or give a counter example. You may use any results we have covered in the course, including the fact that e.g. the language $\{ 0^n 1^n \mid n \in \mathbb{N} \}$ is not regular.

(a) If $A$ is regular and $B \subseteq A$, then $B$ is regular.
(b) If $B$ is regular and $B \subseteq A$, then $A$ is regular.
(c) If $A$ is regular and $B$ is regular, then $A - B$ is regular.
(d) If $A$ is regular and $B$ is not regular, then $A - B$ is not regular.
(e) If $A$ is not regular and $B$ is not regular, then $A \cup B$ is not regular.
(f) If $A$ is not regular and $B$ is not regular, then $A \cap B$ is not regular.

Continues on the other side!
4. (Sipser Problem 1.31) Define the reverse of a language \( A \) as

\[
A^R = \{ w^R \mid w \in A \}.
\]

Show that if \( A \) is regular, then so is \( A^R \).

5. (a) We call a string \( w \) a prefix of a string \( x \), if there is a string \( z \) such that \( x = wz \). For a language \( A \) over alphabet \( \Sigma \), define its prefix language as

\[
\text{PREFIX}(A) = \{ w \in \Sigma^* \mid wz \in A \text{ for some } z \in \Sigma^* \}.
\]

Show that if \( A \) is regular, then so is \( \text{PREFIX}(A) \).

(b) We call a string \( w \) a suffix of a string \( x \), if there is a string \( z \) such that \( x = zw \). For a language \( A \) over alphabet \( \Sigma \), define its suffix language as

\[
\text{SUFFIX}(A) = \{ w \in \Sigma^* \mid zw \in A \text{ for some } z \in \Sigma^* \}.
\]

Show that if \( A \) is regular, then so is \( \text{SUFFIX}(A) \).

6. Let the language \( A_n \) consist of the strings over the alphabet \( \{a, b\} \) where the \( n \)th symbol from the end is a.

(a) Show how \( A_n \) can be recognised by an NFA with at most \( n + 1 \) states.

(b) Show that any DFA for the language \( A_n \) has at least \( 2^n \) states.