1. For both of the automata given below, find an equivalent regular expression using the method explained in the textbook.

![Automata Diagram]

2. Consider the following attempt to prove that the language $A = \{ 0^n1^m \mid n, m \in \mathbb{N} \}$ is not regular:

Suppose for contradiction that $A$ is regular. By the pumping lemma, it has a pumping length $p$. Let $s = 0^p1^p$. Then $s \in A$ and $|s| \geq p$. Split $s$ into parts $s = xyz$ where $x = 0^{p-1}$, $y = 01$ and $z = 1^{p-1}$. Now $xyyz = 0^{p-1}01011^{p-1} \notin A$, so the claim of the pumping lemma is violated.

Is this a valid proof? If not, then what is wrong?

3. We say that $p$ is the minimum pumping length for a language, if it is a pumping length, but any $p'$ with $p' < p$ is not a pumping length. (Notice that if $p$ is a pumping length, then so is any $p' \geq p$.) What are the minimum pumping lengths for the following languages?

- $C_1 = (01)^*$
- $C_2 = 1^*01^*01^*$
- $C_3 = \varepsilon$
- $C_4 = 010 \cup 00100$.

4. Use the pumping lemma (or any other method of your choice) to prove that the following languages are not regular:

(a) $\{ a^m b^n c^n \mid n, m \geq 1 \}$
(b) palindromes over the alphabet $\{ a, b, c \}$
(c) $\{ 0^n1^m \mid n \in \mathbb{N} \}$.

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5. Which of the following languages are regular, which non-regular? (For languages $A_1$ ja $A_2$ the alphabet is $\{0, 1\}$, for the rest $\{a, b, c\}$.)

\[
\begin{align*}
A_1 &= \{ 0^n1^m0^n \mid n, m \in \mathbb{N} \} & A_2 &= \{ 0^n0^n \mid n \in \mathbb{N} \} \\
A_3 &= \{ uw^R \mid w \in \Sigma^* \} & A_4 &= \{ wuw^R \mid w, u \in \Sigma^+ \} \\
A_5 &= \{ wxw^R \mid w \in \Sigma^*, x \in \Sigma \} & A_6 &= \{ abca^n b^n c^n \mid n \in \mathbb{N} \}
\end{align*}
\]

Justify your answers. You may use any known results about regular languages, in particular the previous problem.

6. (Sipser Problem 1.63)

(a) Assume that $A$ is an infinite regular language. Prove that there are infinite regular languages $B_1$ and $B_2$ such that $B_1 \cup B_2 = A$ and $B_1 \cap B_2 = \emptyset$.

(b) Write $A \subset \subset B$ to denote that $A \subset B$ and $B - A$ is infinite. Assume that $B$ and $D$ are regular languages such that $B \subset \subset D$. Prove that there is a regular language $C$ such that $B \subset \subset C$ and $C \subset \subset D$. 