To make grading easier, please write your answers to each of the problems 1, 2, 3 and 4 on its own sheet. Answer to all parts of all the problems. The maximum score is 24 points.

1. [2+2+2+2 points] Give a state diagram for a deterministic or nondeterministic finite automaton for the following languages over the alphabet \{a, b, c\}.

   (a) strings that contain the substring abc
   (b) string where the combined number of symbols a and b is evenly divisible by three.

Give a regular expression for the following languages over the alphabet \(\Sigma = \{a, b, c\}\).

   (c) strings of length at least two where the first two symbols are the same
   (d) strings that do not contain substrings ab or ba.

2. [3+3 points]

   (a) Convert the following NFA into an equivalent DFA by applying the method covered in the course:

   ![NFA Diagram]

   (b) Convert the regular expression \((a^*b \cup c)^*\) into an NFA using the method covered in the course:

   You don’t need to show any intermediate steps assuming one can see from the end result how it was obtained.

3. [2+2+2 points] For arbitrary languages \(A\) and \(B\) over the alphabet \(\{0, 1\}\), which of the following claims are true:

   (a) If \(A\) is regular and \(A \cup B\) is regular, then \(B\) is regular.
   (b) If \(A\) is regular and \(A \cup B\) is not regular, then \(B\) is not regular.
   (c) If \(A\) is regular, \(A \cup B\) is regular and \(A \cap B\) is regular, then \(B\) is regular.

   Justify your answers by giving a proof or a counter example. Keep you arguments short but precise. You may use any properties of regular languages that have been proved in the course.

4. [4 points] Show that the language

   \[ A = \{a^m b^n c^k \mid m = n \text{ or } n = k \} \]

   over the alphabet \(\{a, b, c\}\) is not regular. You may use the pumping lemma and other general properties of regular languages you know from the course, but not results that directly state that a specific language is not regular.