582206 Models of Computation
Separate and renewal examination, 3 February 2012, 16:00–20:00, Exactum A111
examiner: Jyrki Kivinen

Answer all the problems. Maximum score is 60 points.

1. [12 points] For the following languages over alphabet \{a, b, c\}, give both a regular expression and a (deterministic or non-deterministic) finite automaton:
   
   (a) strings that end in bba
   (b) strings in which each 'b' is immediately preceded and immediately followed by 'a' (in other words, 'b' only appears as part of the combination “aba”).

2. [10 points] Transform the following NFA into a DFA. If your transformation is not done by following the method covered in the course, explain briefly what you have done and why the result is correct.

   Give either a regular expression or a short verbal description for the language recognised by the automaton (no justifications needed).

3. [10 points] Define an expression (that represents some limited operations in set theory) as follows:
   - A, B and C are expressions
   - if w is an expression, then so is \((w)\)
   - if v and w are expressions, then so are v \(\cup\) w and v \(\times\) w
   - there are no other expressions.

   Thus, expressions are strings over the alphabet \{A, B, C, \(\cup\), \(\times\), \(), \)\}. Give a context-free grammar for the language that consists of all such expressions. Give for the expression \(A \cup B \times C\) all the parse trees and the corresponding leftmost derivations according to your grammar.

Continues on the reverse side!
4. [12 points] We say that a language $A$ includes its own prefixes if $\text{Prefix}(A) \subseteq A$, in other words if for all strings $x \in A$ in the language, also all the prefixes of $x$ are in $A$.

(a) It is true that any regular language include its own prefixes? It it true that if a language includes its own prefixes, it is regular?

(b) Is the class of languages that include their own prefixes closed under the union operation? How about under the concatenation operation?

In each case, justify your answer precisely. You may use any results that have been proven in the course.

5. [16 points]

(a) What is the Church-Turing thesis? Explain briefly also some justifications that have been given for the thesis.

(b) Formulate as formal language the computational problem

\textbf{Given:} a Turing machine $M$, a state $q$ of $M$, a string $w$ over the input alphabet of $M$

\textbf{Question:} does $M$ on input $w$ enter state $q$ at least once.

Is your formal language decidable? Is it recognisable? Justify your answers. You may use the known results about decidability of the halting problem etc.