1. [12 points] Give a regular expression and either a DFA or an NFA for both of the following languages:

   (a) The language $A_1$ consists of all strings over the alphabet $\{a, b, c\}$ that have length at least three and end either in “abc” or in “cba”.

   (b) The language $A_2$ consists of all strings over the alphabet $\{0, 1\}$ where the number of zeros is divisible by 3.

2. [8 points] Let us define a list recursively as follows:

   - There is an empty list, which is denoted by NIL.
   - Given $n$ lists $L_1, \ldots, L_n$ for some $n \geq 1$, we can construct a list $(L_1, \ldots, L_n)$.
   - There are no other lists besides what the above implies.

   Thus, the following are examples of a list:

   $$((((\text{NIL}))))$$

   $$(\text{NIL}, (\text{NIL}), (\text{NIL}, (\text{NIL})), (\text{NIL}, (\text{NIL}), (\text{NIL}, (\text{NIL}))))$$

   $$(\text{NIL}, \text{NIL}, \text{NIL}, \text{NIL}).$$

   We can represent a list as a string over the alphabet that consist of the symbols ”NIL”, ”,” (comma) and the paranteses “(“ and ”)”. Give a context-free grammar that generates all such representations of lists.

3. [8 points] Let $C$ be the language generated by the grammar

   $$S \rightarrow aSb | bSa | SS | \varepsilon.$$ 

   Convert $C$ into an equivalent push-down automaton using the method given in the textbook. Explain in English, which strings belong to $C$.

4. [12 points] Let the language $D$ over the alphabet $\{0, 1\}$ consist of strings where the number of zeros exceeds the number of ones exactly by 3. The order of zeros and ones does not matter. Thus, the language $D$ includes e.g. the strings 000, 001010100 and 1111100000000. Prove that $D$ is not a regular language. You may take as known any results given in the textbook.

5. [10 points] What is meant by the Church-Turing thesis? What arguments can be given to support the thesis? What implications does the thesis have for research in computer science?

6. [10 points] Prove that if a language $A$ and its complement $\overline{A}$ are both Turing-recognisable, then $A$ is decidable.