582206 Models of Computation
Separate examination, 11 September 2012, 16:00–20:00, Exactum A111
examiner: Jyrki Kivinen

You should answer all the problems. Maximum score is 60 points.

1. [4 + 4 + 4 points]

   (a) The language \( A_1 \) over the alphabet \{a, b, c\} consists of all the strings where there is always at least one symbol 'b' between any two symbols 'a'. Give either a regular expression or a deterministic finite automaton for \( A_1 \) (whichever you prefer).

   (b) The language \( A_2 \) over the alphabet \{a, b, c\} consists of all the strings in which the substring abc does not occur. Give either a regular expression or a deterministic finite automaton for \( A_2 \) (whichever you prefer).

   (c) Give a context-free grammar for the language \( A_3 = \{a^n b^m | n \geq 0, m \geq 2n\}\) over the alphabet \{a, b\}.

2. [6 + 6 points]

   (a) Convert the regular expression \( 1^*(001 \cup 101)^* \) into a non-deterministic finite automaton using the method explained on the course.

   (b) Convert the non-deterministic automaton given below into a deterministic one using the method explained on the course.

   Give either a short verbal description or a regular expression for the language recognised by the automaton (no justification needed).

   ![Automaton Diagram]

   For both parts, if it can clearly be seen from the end result that the conversion has been made following the method given on the course, no additional explanations are needed.

3. [4 + 4 + 4 points] For arbitrary languages \( A \) and \( B \) over the alphabet \{0, 1\}, which of the following claims are true:

   (a) If \( A \) is regular and \( A \cup B \) is regular, then \( B \) is regular.

   (b) If \( A \) is regular and \( A \cup B \) is not regular, then \( B \) is not regular.

   (c) If \( A \) is regular, \( A \cup B \) is regular and \( A \cap B \) is regular, then \( B \) is regular.

   Justify your answers by giving a proof or a counter example. Keep you arguments short but precise. You may use any properties of regular languages that have been proved in the course.

Continues on the other side!
4. [12 points] Given a context-free grammar $G$ with terminal alphabet $\Sigma = \{ a, b, c \}$, the problem is to decide whether $L(G)$ includes at least one string that contains no other symbols than ‘a’. (The empty string also qualifies for this.) Give an algorithm to solve the problem.

5. [12 points] As a part of a programming course, the students have to write a Java program that output the numbers 1, \ldots, 10 and then halts.

   (a) Is it possible to write a program that automatically checks all the solutions turned in by the students and tells, which of them work correctly and which don’t?

   (b) Is it possible to write a helper program that finds from among the students’ programs all those that go into an eternal loop and never output anything?

Justify your answers. Pay attention to how the theoretical concepts introduced on the course are (or are not) applicable to modelling the situation.

In both parts the checking program is required to never make a mistake, even if a student knows its source code and tries to mislead it in either direction.