1. Let $C(n)$ be the probability that a random graph $G_{n,p}$ includes a triangle (a clique of size 3), when $p = f(n)$.
   
   (a) Show that if $f(n) = o(1/n)$, then $\lim_{n \to \infty} C(n) = 0$.
   
   (b) Show that if $f(n) = \omega(1/n)$, then $\lim_{n \to \infty} C(n) = 1$.

2. A process sends two service requests simultaneously, one to server $A$ and one to server $B$. The reply time of server $A$ is exponentially distributed with parameter $\alpha$ and the reply time of server $B$ exponentially distributed with parameter $\beta$, and the reply times are independent. What is the expected time until both $A$ and $B$ have replied?

3. Remember that a vertex cover of a graph $G = (V,E)$ is any set $I \subseteq V$ such that $\{u,v\} \cap I \neq \emptyset$ for all $(u,v) \in E$. Construct a Markov chain that has as its unique stationary distribution the uniform distribution over all the vertex covers of a given graph. Use results known from the course to demonstrate that your chain does have the desired property.

Maximum score 8 + 8 + 8 = 24.

P.S. Please remember to fill in the course feedback questionnaire on the web!