Problems 1 and 2 are perhaps a bit more challenging; they are intended for solving in groups. In addition to these problems, Round 4 in TRAKLA2 is about heaps (n.b. min-heaps) and could be done at this time.

1. Consider an $m \times n$ matrix $A$, where each element is either a natural number or $\infty$.

We say that $A$ is a Young tableau if each row is in increasing order from left to right and each column is in increasing order from top to bottom. The value $\infty$ is interpreted as larger than any natural number. For example, the natural numbers 2, 5, 7, 11, 15, 16, 21 and 27 could be placed into a $3 \times 4$ Young tableau as follows:

\[
\begin{array}{cccc}
2 & 5 & 11 & 27 \\
7 & 15 & 16 & \infty \\
21 & \infty & \infty & \infty \\
\end{array}
\]

Give an algorithm that removes the smallest element from a Young tableau, replaces it with $\infty$ and then fixes the table so that it again becomes a Young tableau. Thus, one possible result starting from the previous tableau is the following:

\[
\begin{array}{cccc}
5 & 11 & 16 & 27 \\
7 & 15 & \infty & \infty \\
21 & \infty & \infty & \infty \\
\end{array}
\]

Your algorithm should run in time $O(m + n)$.

*Hint:* Think about HEAPIFY for min-heaps.

2. Consider again Young tableaus. We are given as input an $m \times n$ Youngin tableau $A$ and a natural number $k$. The problem is to decide whether $k$ is in the tableau $A$ or not. Give an algorithm that runs in time $O(m + n)$.

3. Write the HEAPIFY algorithm for max heaps in iterative form (that is, without recursion).