

Heuristic search

AND/OR Branch-and-Bound search

Kustaa Kangas

October 17, 2013

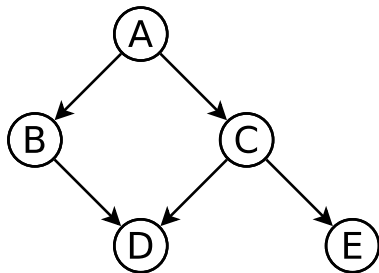
AND/OR search for probabilistic inference

Radu Marinescu, Rina Dechter, 2009

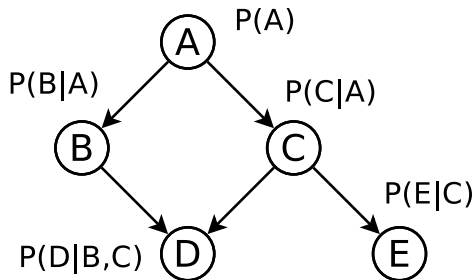
AND/OR Branch-and-Bound Search for Combinatorial Optimization in Graphical Models

- 1 Bayesian networks
- 2 Most Probable Explanation problem
- 3 OR search
- 4 AND/OR search (with Branch-and-Bound)

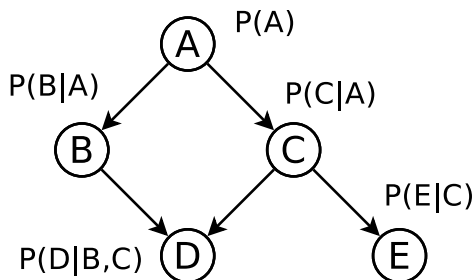
OR search



OR search

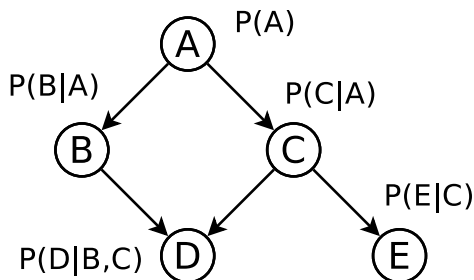


OR search



$$P(A,B,C,D,E) = P(A) \times P(B|A) \times P(C|A) \times P(D|B,C) \times P(E|C)$$

OR search

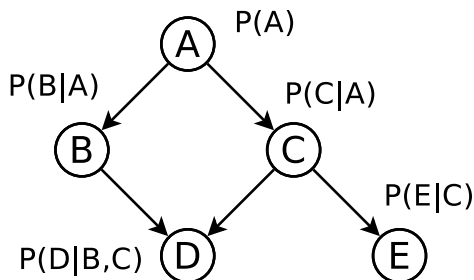


$$P(A,B,C,D,E) = P(A) \times P(B|A) \times P(C|A) \times P(D|B,C) \times P(E|C)$$

Most Probable
Explanation

$$\operatorname{argmax}_{A,B,C,D,E} P(A,B,C,D,E) = ?$$

OR search

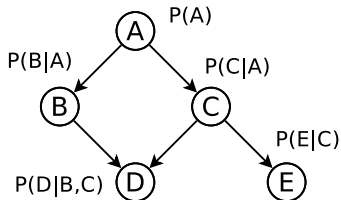


$$P(A,B,C,D,E) = P(A) \times P(B|A) \times P(C|A) \times P(D|B,C) \times P(E|C)$$

Most Probable
Explanation

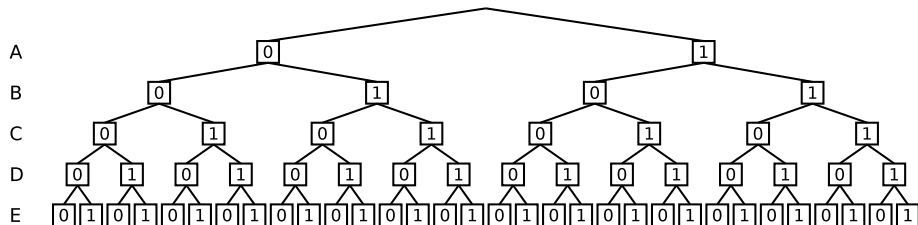
$$\operatorname{argmax}_{A,B,C} P(A,B,C \mid D=\text{true}, E=\text{false})$$

OR search

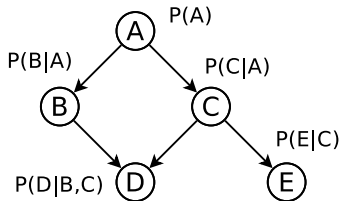


$A, B, C, D, E \in \{0, 1\}$

$\operatorname{argmax}_{A, B, C, D, E} P(A, B, C, D, E) = ?$

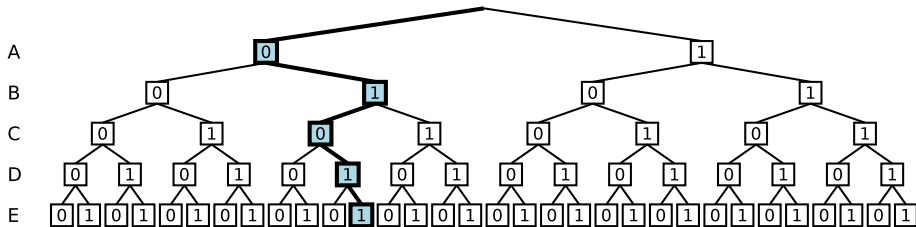


OR search



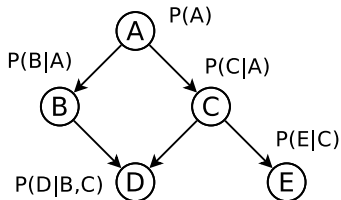
$A, B, C, D, E \in \{0, 1\}$

$\operatorname{argmax}_{A, B, C, D, E} P(A, B, C, D, E) = ?$



$$P(A=0, B=1, C=0, D=1, E=1) = P(A=0) \times P(B=1|A=0) \times P(C=0|A=0) \times P(D=1|B=1, C=0) \times P(E=1|C=0)$$

OR search

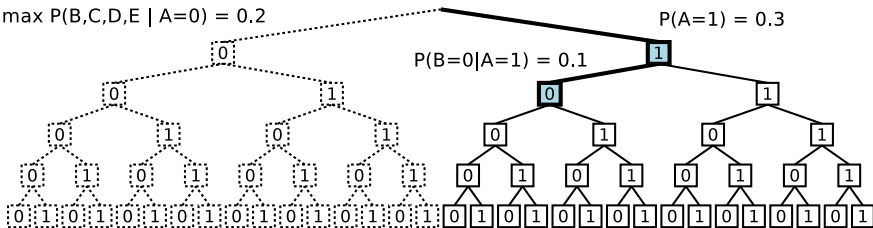


$A, B, C, D, E \in \{0, 1\}$

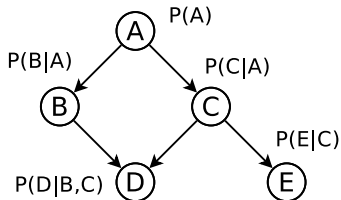
$\operatorname{argmax}_{A, B, C, D, E} P(A, B, C, D, E) = ?$

$\max P(B, C, D, E \mid A=0) = 0.2$

A
B
C
D
E



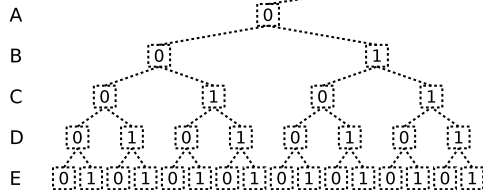
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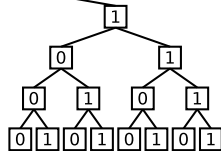
$A, B, C, D, E \in \{0,1\}$

$\operatorname{argmax}_{A, B, C, D, E} P(A, B, C, D, E) = ?$

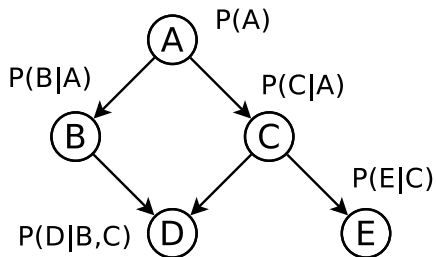
$\max P(B, C, D, E \mid A=0) = 0.2$



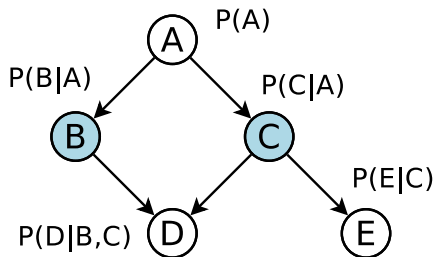
$P(A=1) = 0.3$



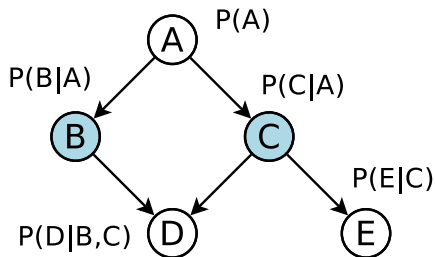
Structural independencies



Structural independencies

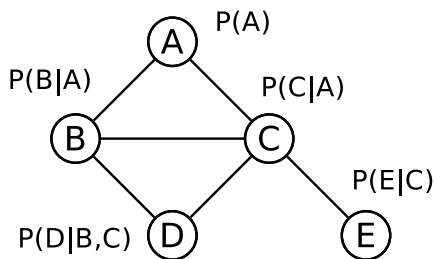


Structural independencies



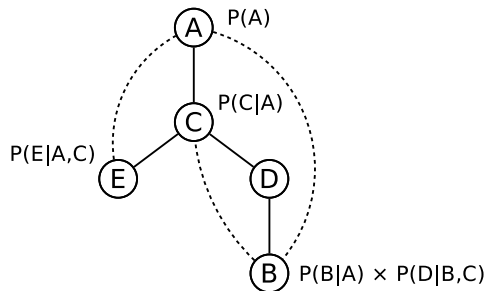
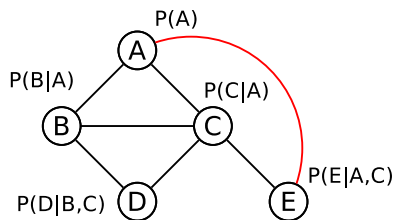
A $P(A), P(B|A), P(C|A)$
D $P(D|B,C)$
E $P(E|C)$

Structural independencies

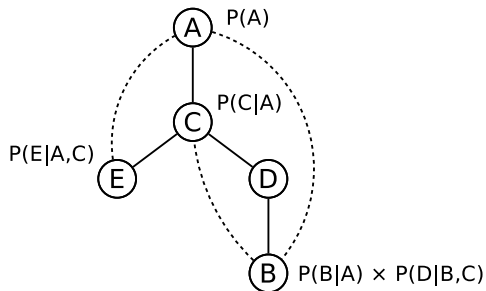
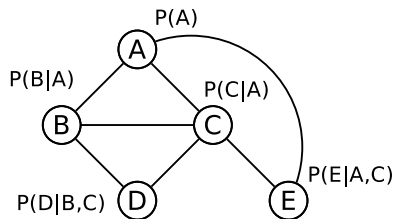


Dependency graph

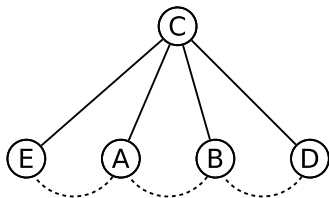
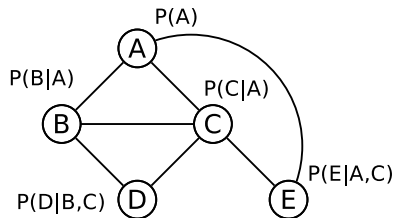
Pseudo tree



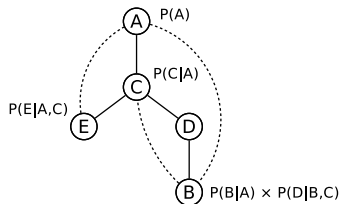
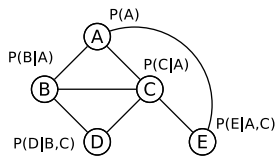
Pseudo tree



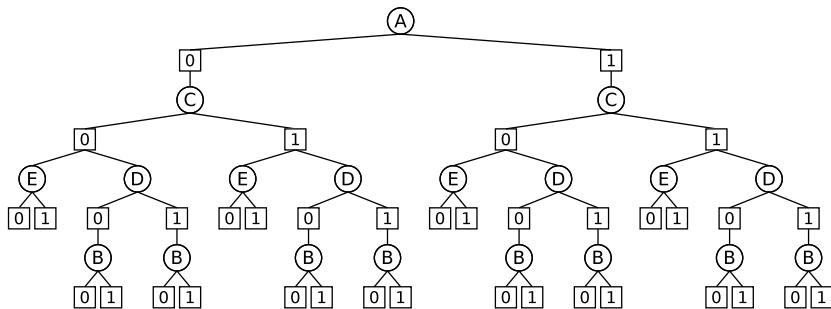
Pseudo tree



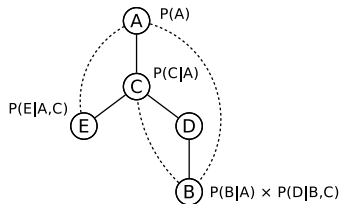
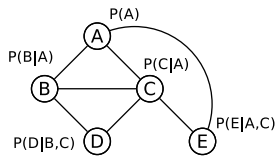
AND/OR search tree



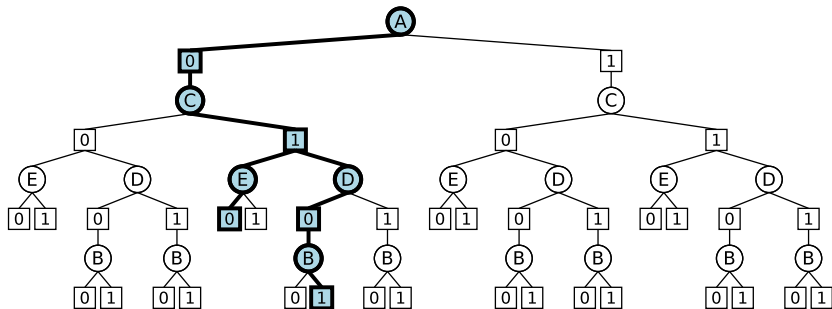
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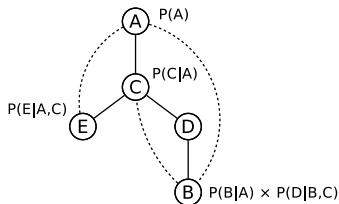
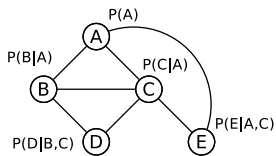
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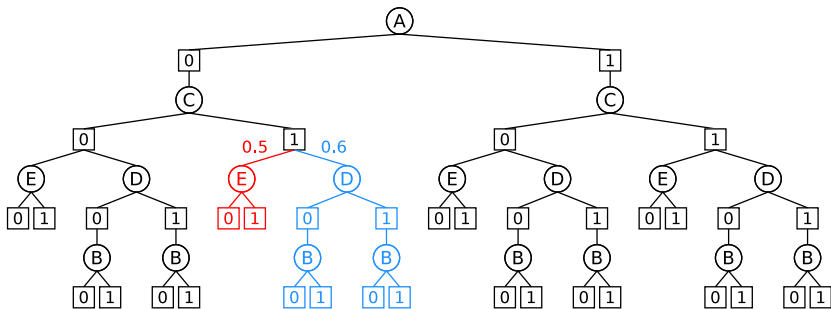
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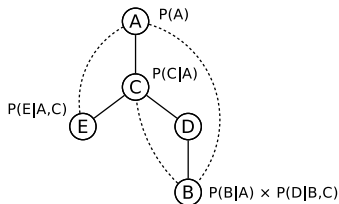
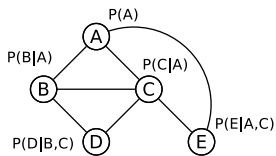
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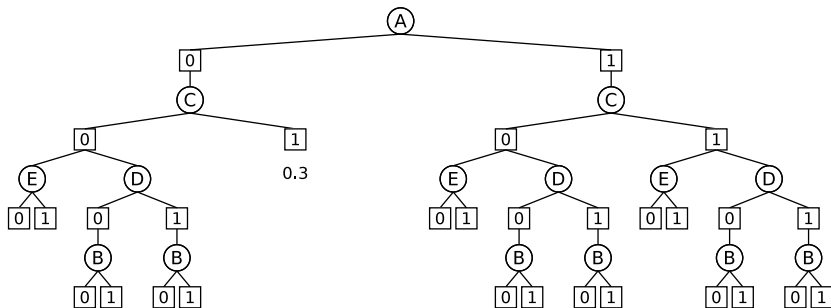
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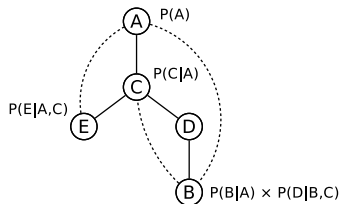
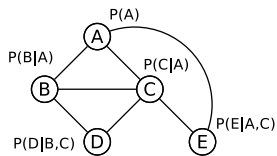
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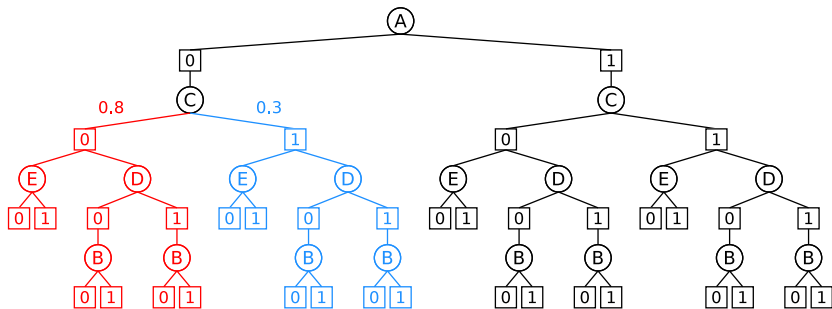
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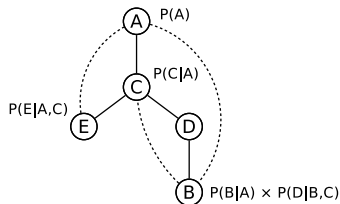
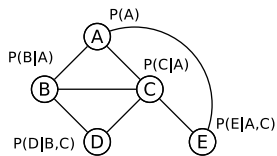
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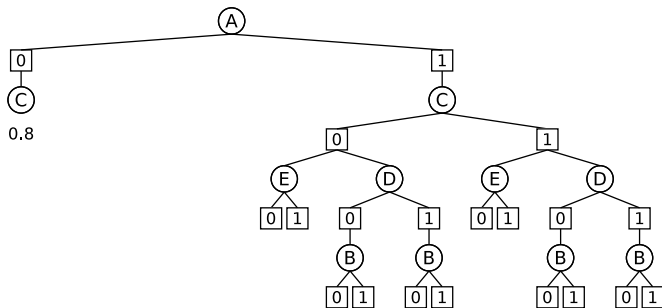
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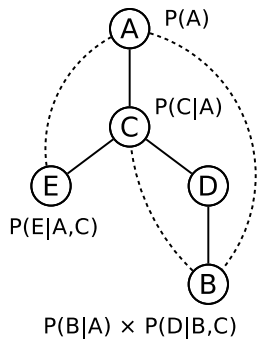
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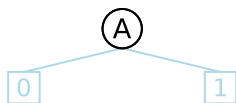
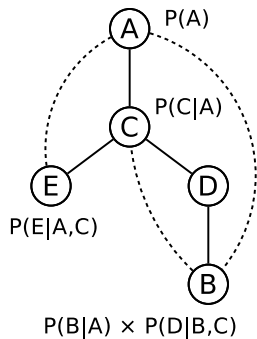


AND/OR search

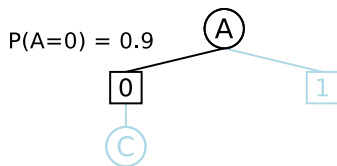
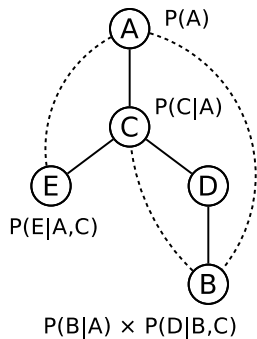


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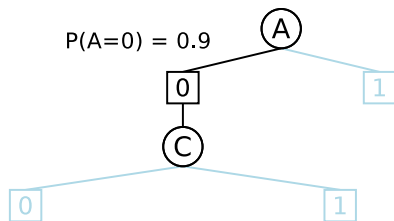
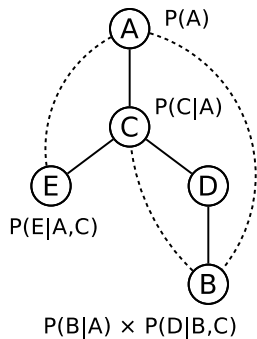
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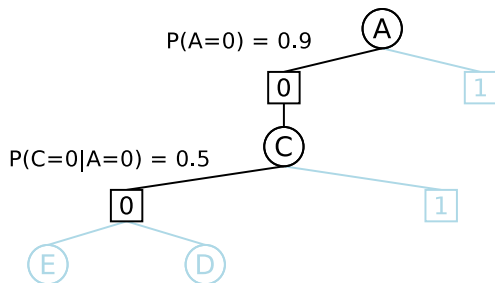
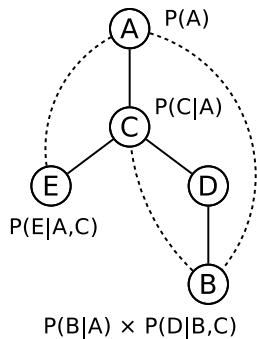
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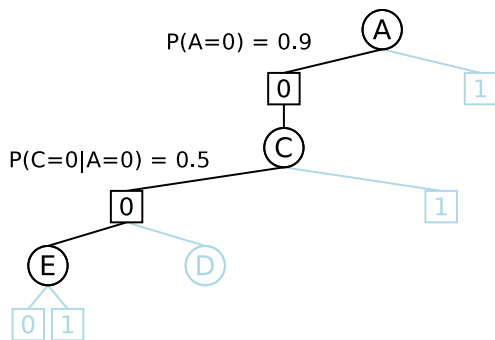
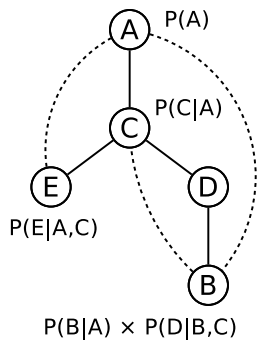
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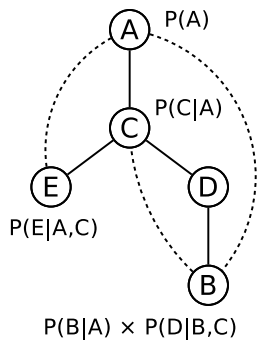
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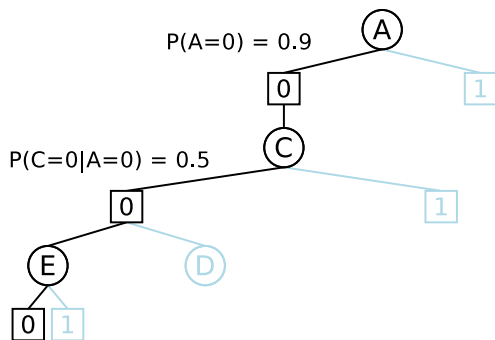
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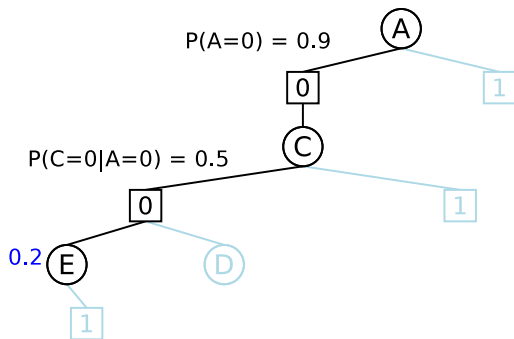
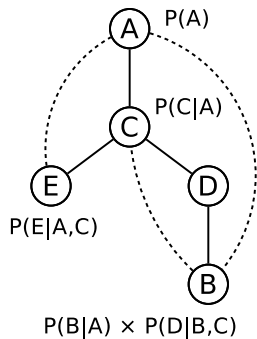
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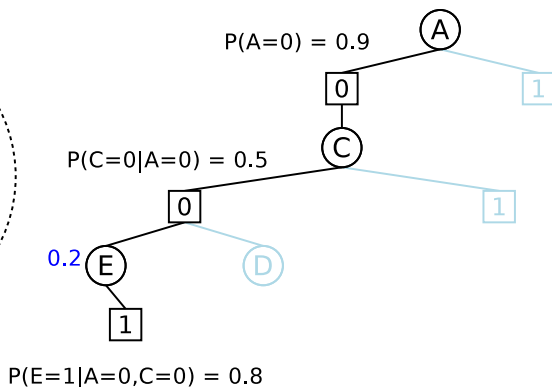
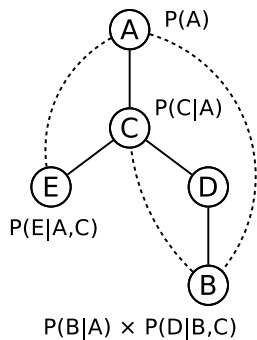
$$P(E=0|A=0,C=0) = 0.2$$



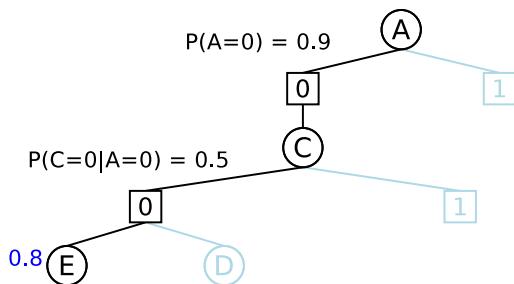
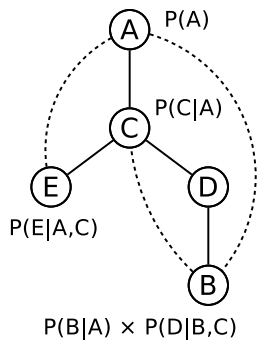
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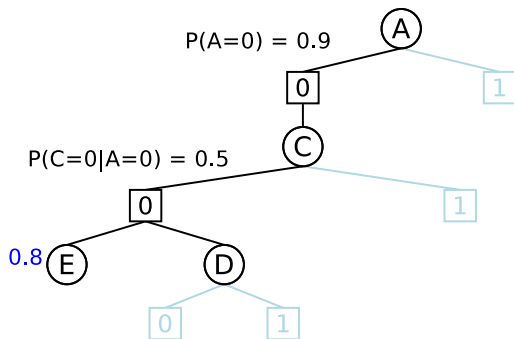
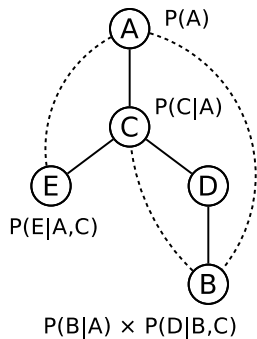
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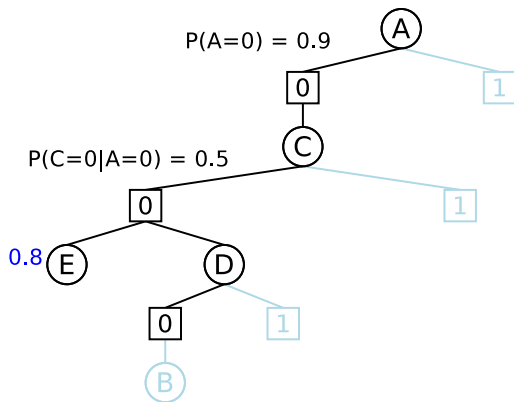
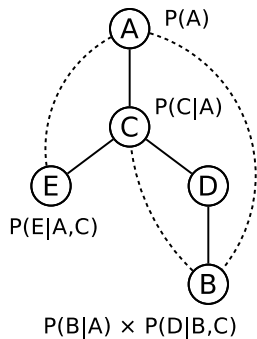
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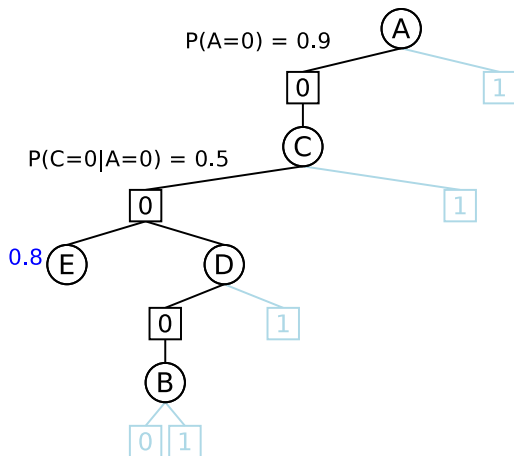
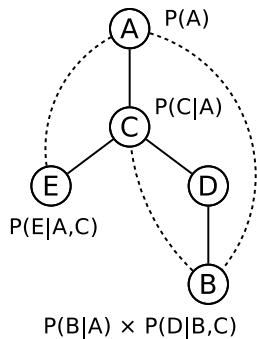
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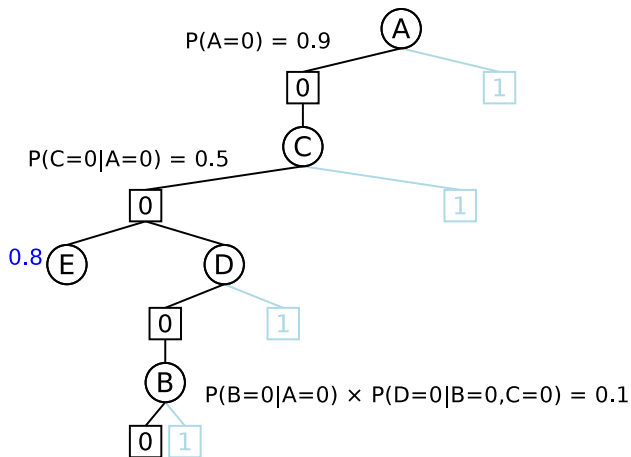
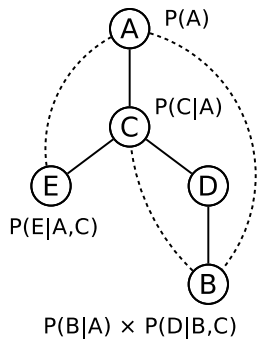
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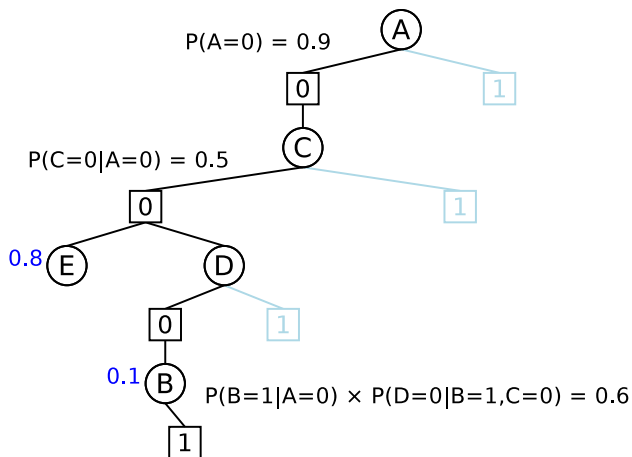
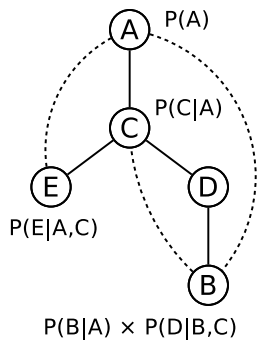
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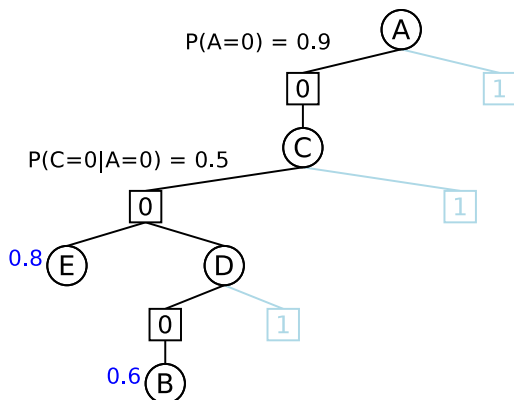
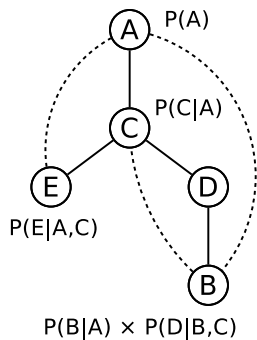
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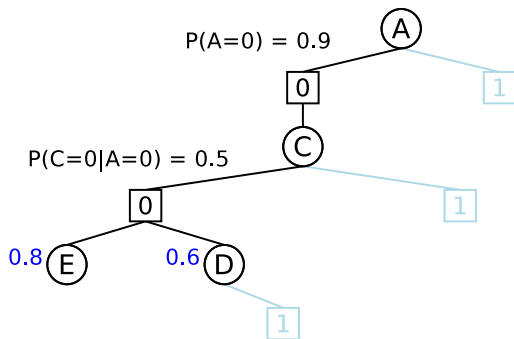
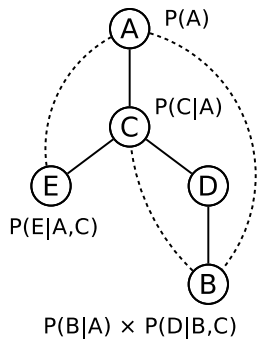
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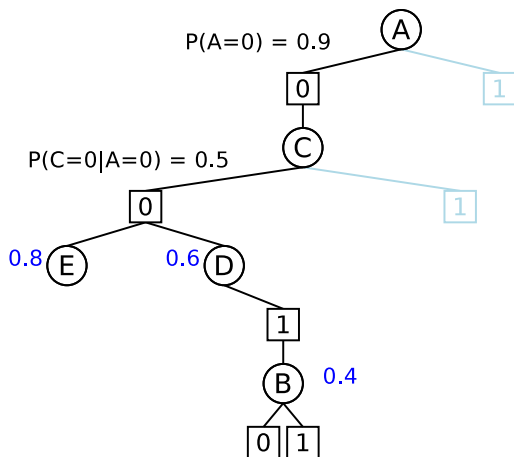
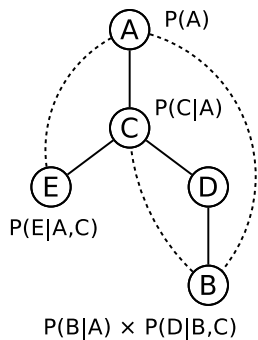
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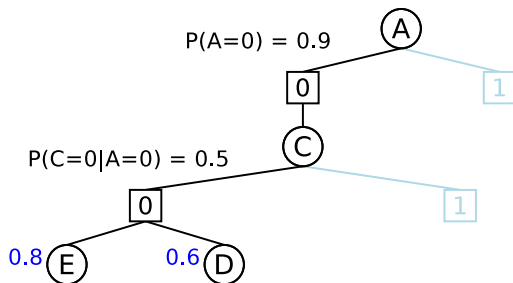
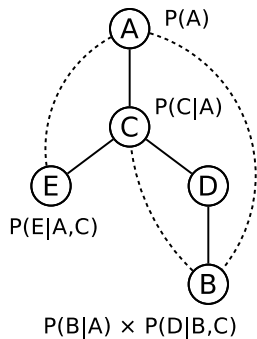
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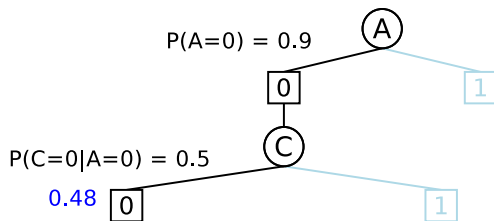
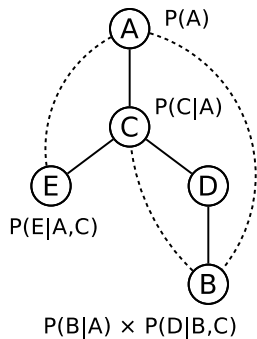
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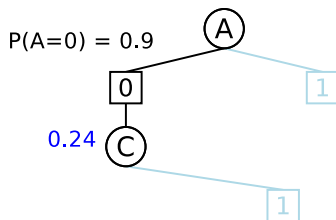
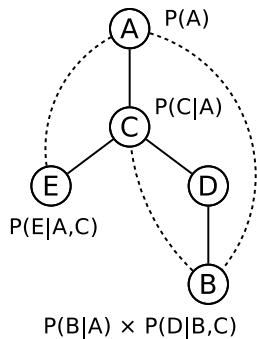
AND/OR search



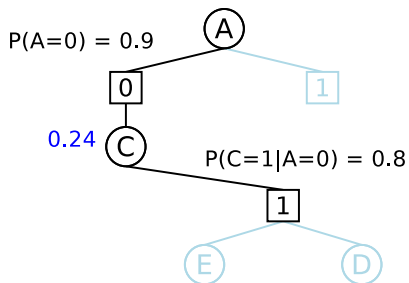
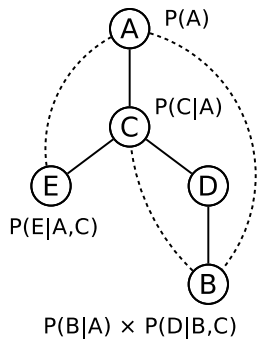
AND/OR search



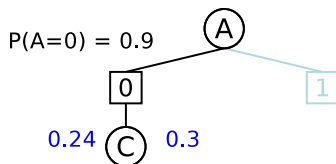
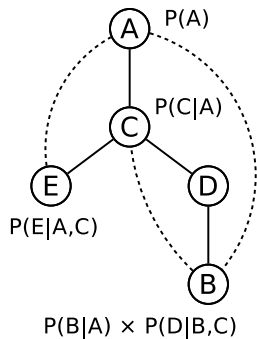
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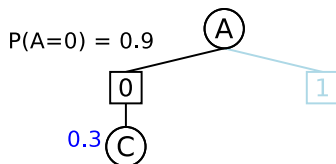
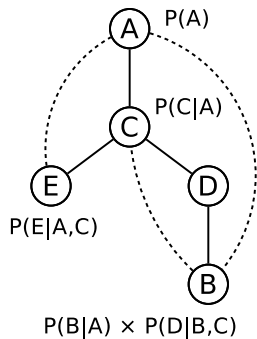
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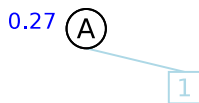
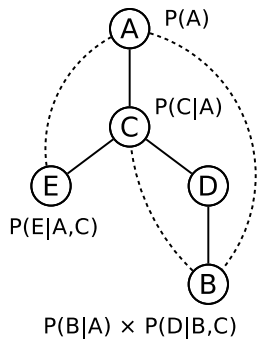
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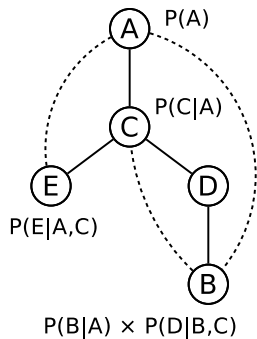
AND/OR search



AND/OR search

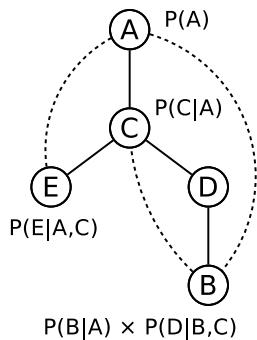


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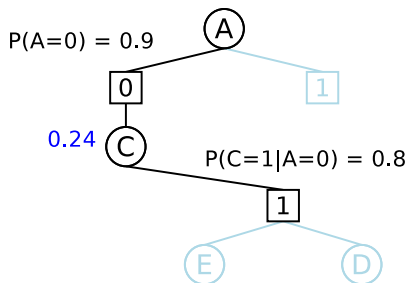
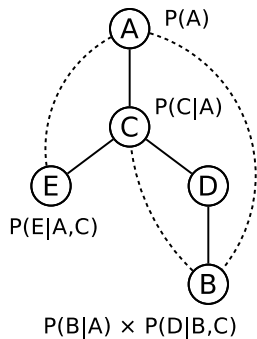
0.27 (A) 0.15

AND/OR search

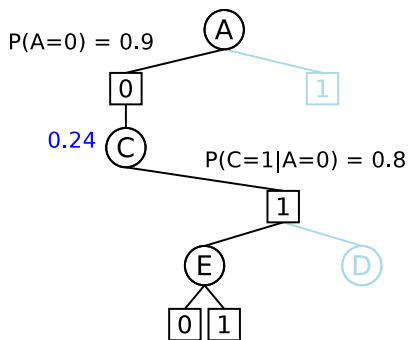
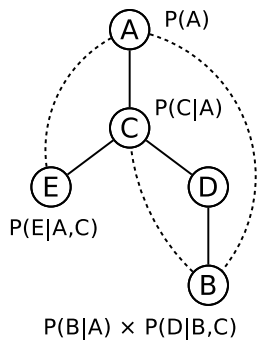


0.27

AND/OR search

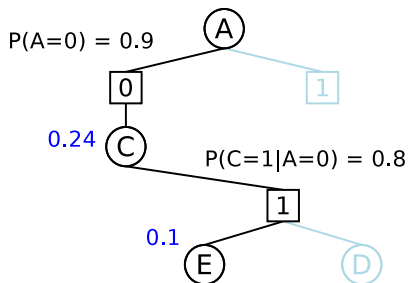
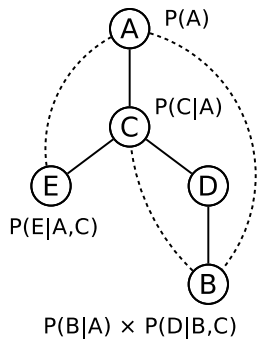


AND/OR search

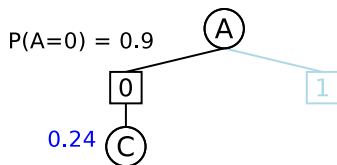
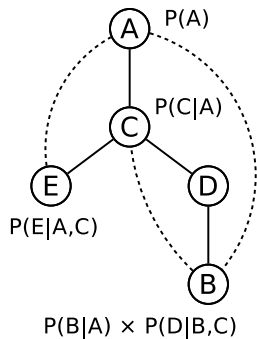


$$P(E=0|A=0,C=1) = 0.1$$
$$P(E=1|A=0,C=1) = 0.05$$

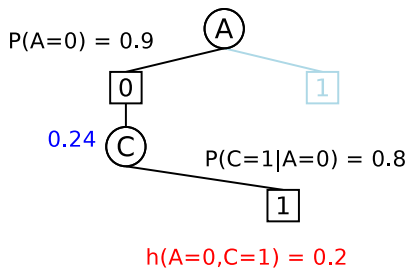
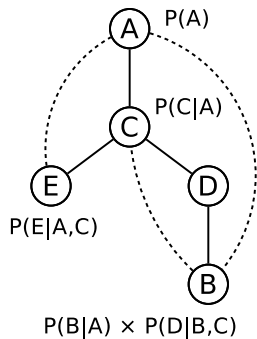
AND/OR search



AND/OR search



AND/OR search



Conclusion

- The MPE can be formalized as a search problem.
- OR search instantiates variables sequentially.
- AND/OR search splits into independent subproblems.
- Both search spaces may be pruned with Branch-and-Bound.

We didn't cover

- Heuristic functions
- How to find pseudo trees?

Questions?