Heuristic search

Weighted A*

Kustaa Kangas

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Weighted A*

Weighted A* search – unifying view and application

Rüdiger Ebendt, Rolf Drechsler, 2009

Weighted A*

- Weight the heuristic to quickly direct the search.
- Save time, get bounded suboptimality in exchange.

Outline

1. Three approaches: WA*, DWA*, A^*_ε
2. Unifying view
3. Monotone heuristic
4. Approximate BDD minimization
5. Experiments

Standard A*

Standard A*

$$f(q) = g(q) + h(q)$$

Finds an optimal path if h is admissible, i.e. $h(q) \leq h^*(q)$.

Constant inflation

WA*: constant inflation

$$f^{\uparrow}(q) = g(q) + (1+\varepsilon)h(q)$$

where $\varepsilon \geq 0$.

If h is admissible, then WA* is ε-admissible, i.e.

$$g(q) \leq (1+\varepsilon)C^*$$

for all expanded q where C^* is the length of an optimal path.

ε-admissibility

If h is admissible, then WA* is ε-admissible.

Proof.

Let $s \ldots q \ldots t$ be an optimal path where q is the first node of the path in the open list. Assume a goal state t is expanded. This can happen only if

$$\begin{aligned} g(t) = f(t) &\leq f(q) \\ &= g(q) + (1+\varepsilon)h(q) \\ &\leq g^*(q) + (1+\varepsilon)h^*(q) \\ &\leq (1+\varepsilon)(g^*(q) + h^*(q)) \\ &= (1+\varepsilon)C^* \end{aligned}$$

$\square$

Dynamic weighting

DWA*: Dynamic weighting

$$f^{DW}(q) = g(q) + \left(1 + \varepsilon \cdot \left[1 - \frac{d(q)}{N}\right]\right) h(q)$$

where

$d(q)$	depth of q
N	depth of optimal solution

Idea: as the search goes deeper, emphasize the heuristic less.

How do we get N?

- Sometimes known beforehand: e.g. BDD minimization.
- Generally not known: use an upper bound.

A^*_ε

Keep the original cost function

$$f(q) = g(q) + h(q)$$

Instead of expanding q with the smallest $f(q)$, define

$$FOCAL = \left\{ q \in OPEN \mid f(q) \leq (1+\varepsilon) \cdot \min_{r \in OPEN} f(r) \right\}$$

Use *another* heuristic h_F to choose a minimum from FOCAL, i.e.

$$\hat{q} = \arg\min_{q \in FOCAL} h_F(q)$$

A^*_ε

$$f(q) = g(q) + h(q)$$

$$\hat{q} = \operatorname*{arg\,min}_{q \in FOCAL} h_F(q)$$

Original idea:

- h estimates solution cost
- h_F estimates remaining search effort

Suggestions for h_F:

- $h_F = h$
- $h_F(q) = N - d(q)$

Unifying view

$$f(q) = g(q) + h(q)$$

$$FOCAL = \left\{ q \in OPEN \mid f(q) \leq (1+\varepsilon) \cdot \min_{r \in OPEN} f(r) \right\}$$

WA* and DWA* are actually special cases of A^*_ε

$$h_F(q) = f^{\uparrow}(q) = g(q) + (1+\varepsilon)h(q) \quad \text{WA}^*$$

$$h_F(q) = f^{DW}(q) = g(q) + \left(1 + \varepsilon \cdot \left[1 - \frac{d(q)}{N}\right]\right) h(q) \quad \text{DWA}^*$$

Unifying view

A^*_ε is a unifying framework.

- Any result for A^*_ε follows for WA^* and DWA^*
  - e.g. ε-admissibility
- Makes the approaches comparable (same f)

Unifying view

Concern: what if weighted A^* expands many q with

$$C^* \leq f(q) \leq (1+\varepsilon)C^*$$

- Could overcome the advantages of directing the search.
- General A^*_ε makes no guarantees.
- For WA^* and DWA^* this happens relatively rarely.

Monotone heuristic

Monotone heuristic

If for every state q and its descendant q'

$$h(q) \leq c(q, q') + h(q')$$

the heuristic is *monotone* or *consistent*.

A* with a monotone heuristic

- When q is expanded, $g(q) = g^*(q)$
- Expanded states are never reopened

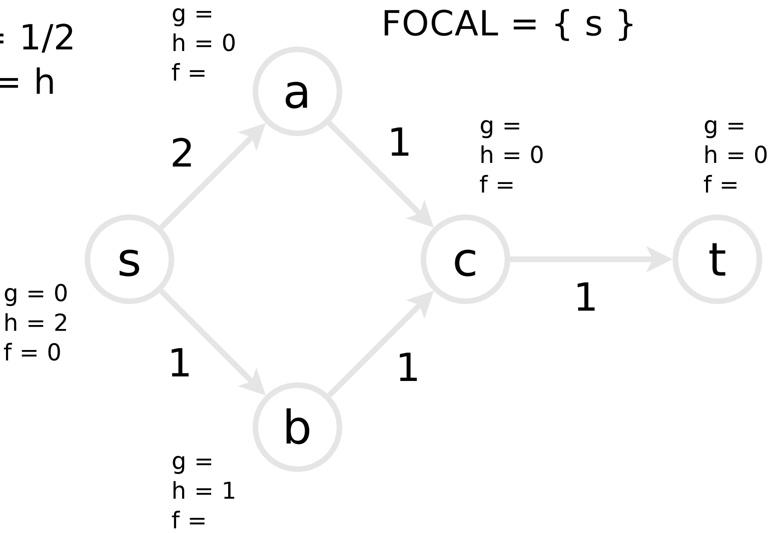
Does this hold for weighted A*?

Monotone heuristic

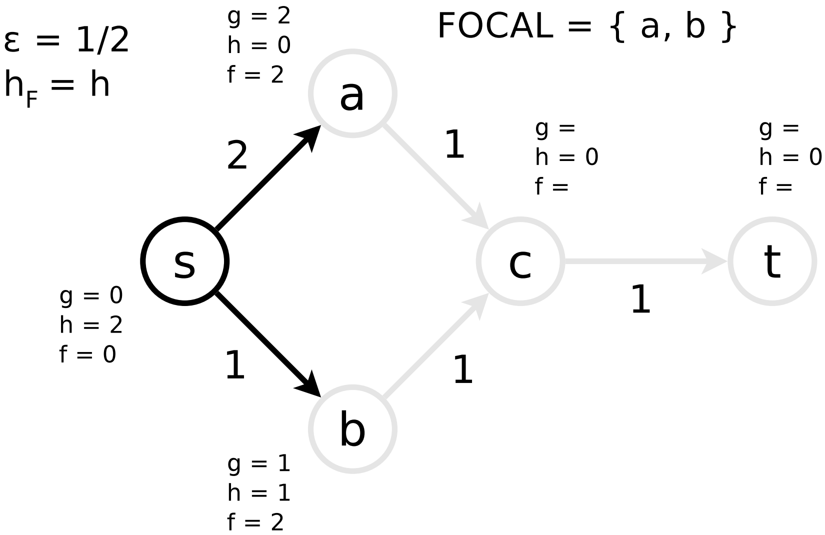
$\varepsilon = 1/2$

$h_F = h$

FOCAL = { s }



Monotone heuristic



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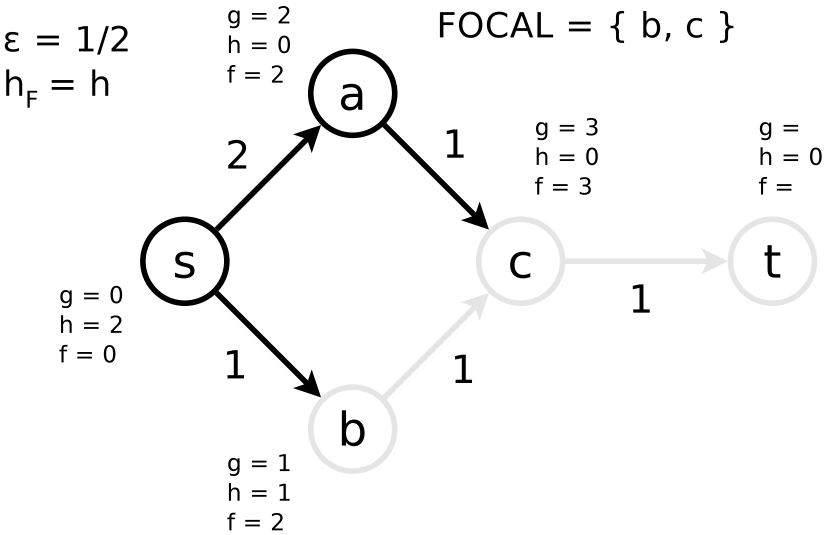

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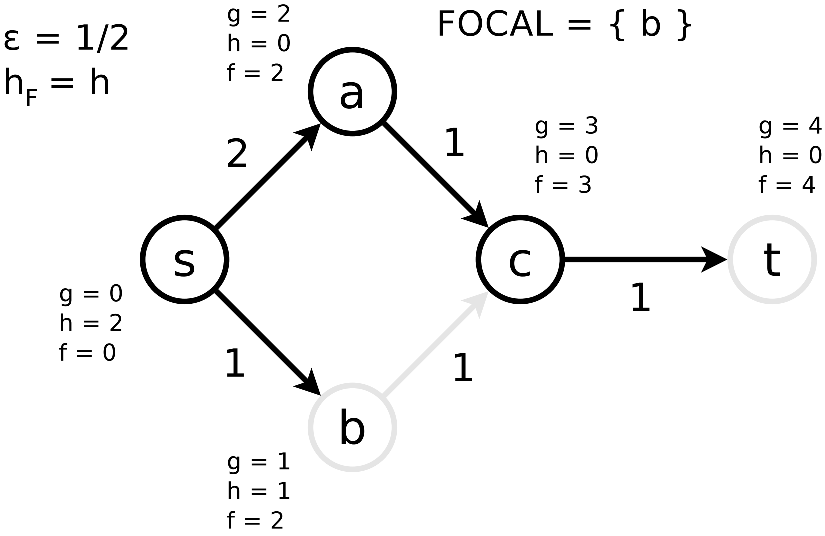

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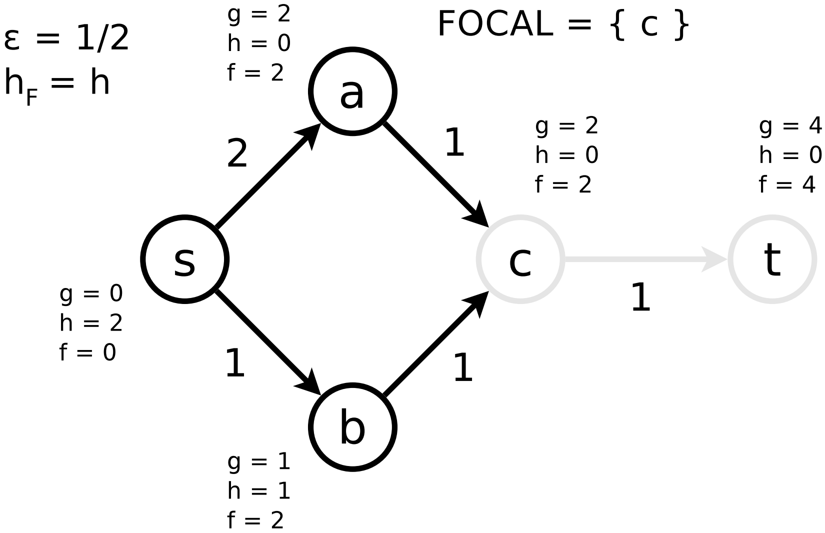

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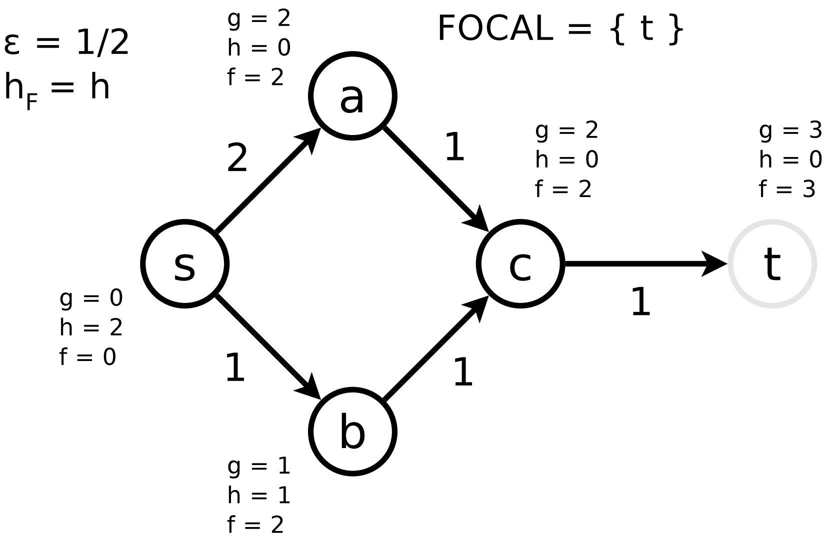
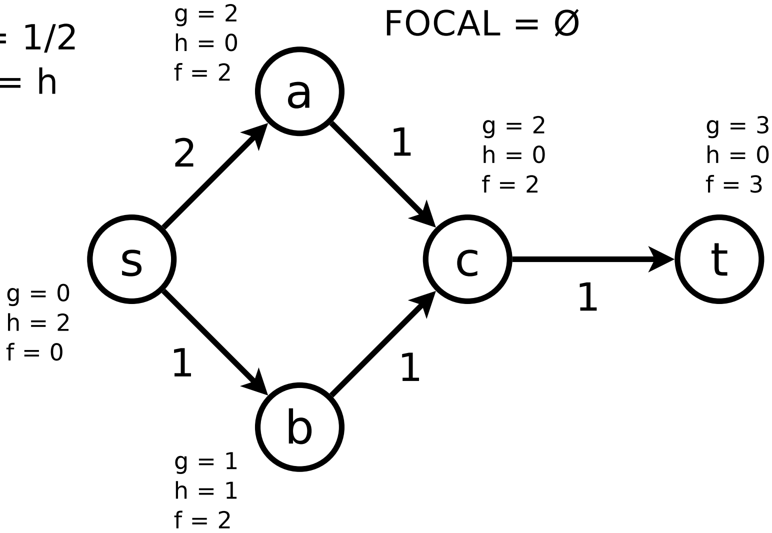


Monotone heuristic

$\varepsilon = 1/2$

$h_F = h$

FOCAL = Ø



Monotone heuristic

Turns out no. However, we do get the bound

$$g(q) \leq (1+\varepsilon)g^*(q) + \varepsilon \cdot h(q)$$

for all expanded q.

- Weighted A* benefits less from a monotone heuristic.
- Reopening may increase running times significantly.

Without reopening

What if we don't reopen states? Simply ignore any new better path.

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- Intuition: shortcutting requires always at least two states.
- Each shortcut accumulates the error by a factor of $(1+\varepsilon)$.
- WA* and DWA* are still ε-admissible without reopening.

Experiments

All variants were evaluated on a number of problems:

- BDD minimization
- Blocksworld
- Sliding-tile puzzle
- Depots
- Logistics
- PSR
- Satellite
- Freecell
- Driverlog

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Boolean functions

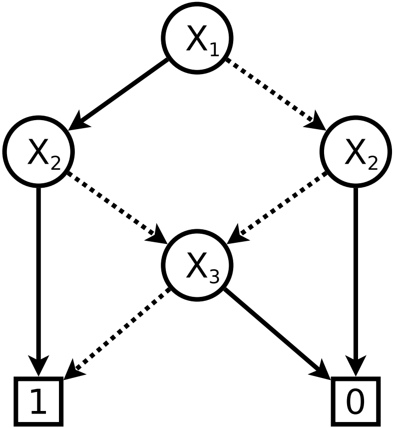
A Boolean function is a function

$$f : \{0,1\}^n \to \{0,1\}$$

Can be represented as a table, e.g. $n = 3$:

X_1	X_2	X_3		Y
0	0	0	$\mapsto$	1
0	0	1	$\mapsto$	0
0	1	0	$\mapsto$	0
0	1	1	$\mapsto$	0
1	0	0	$\mapsto$	1
1	0	1	$\mapsto$	0
1	1	0	$\mapsto$	1
1	1	1	$\mapsto$	1

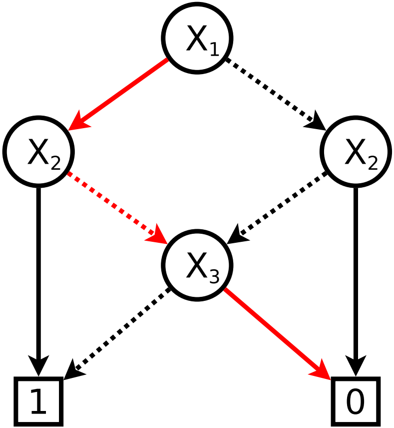
Boolean decision diagrams



Boolean decision diagrams

$X_1 = 1$
$X_2 = 0$
$X_3 = 1$

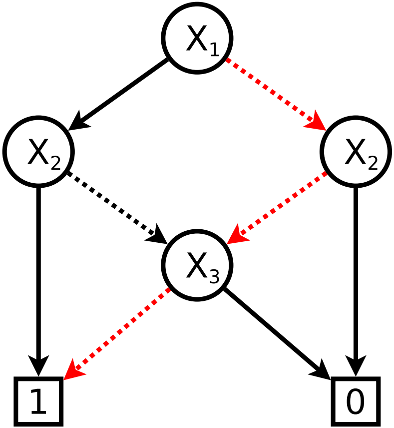
$Y = 0$



Boolean decision diagrams

$X_1 = 0$
$X_2 = 0$
$X_3 = 0$

$Y = 1$



Boolean decision diagrams

Several applications

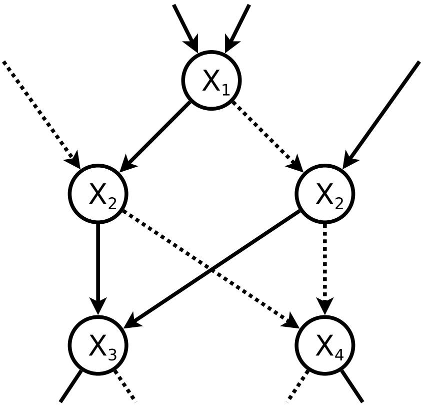
- Model checking
- Sparse-memory applications
- Planning
- Symbolic heuristic search
- Enchancing heuristic search (e.g. A^*)

In general we want BDDs to be as small as possible.

- Easier to read
- Take less memory
- Faster to evaluate

Boolean decision diagrams

BDDs are not unique and can often be simplified.



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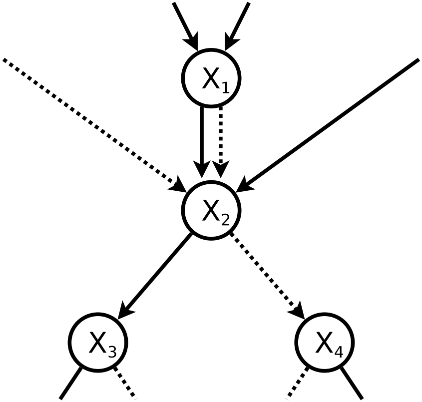

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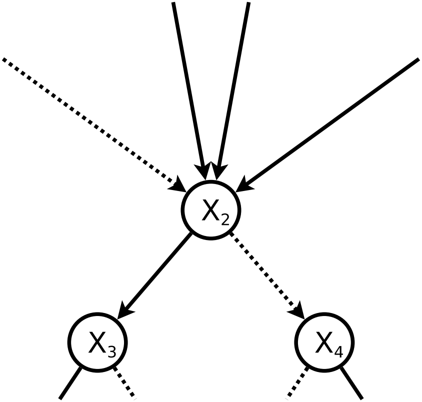
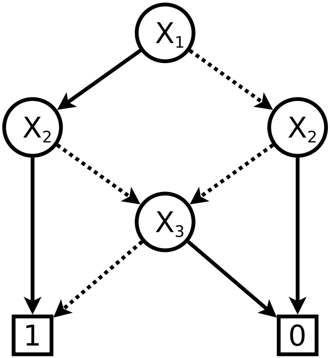


Boolean decision diagrams

- For a fixed permutation of variables, applying merge and deletion iteratively yields a minimal BDD.
- However, the permutation determines how small BDDs we can achieve.

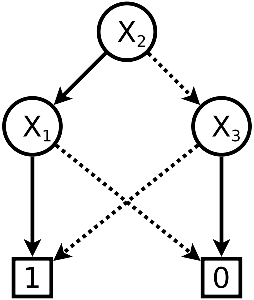
Boolean decision diagrams

Optimal BDD for permutation X_1, X_2, X_3.



Boolean decision diagrams

Optimal BDD for permutation X_2, X_1, X_3 (and X_2, X_3, X_1)



Boolean decision diagrams

BDD minimization problem: find an ordering of variables that yields a minimal BDD (least nodes)

- NP-hard (decision version is NP-complete)
- Can be solved exactly in $O(3^n n)$.
- Can often be solved fast with heuristic search.

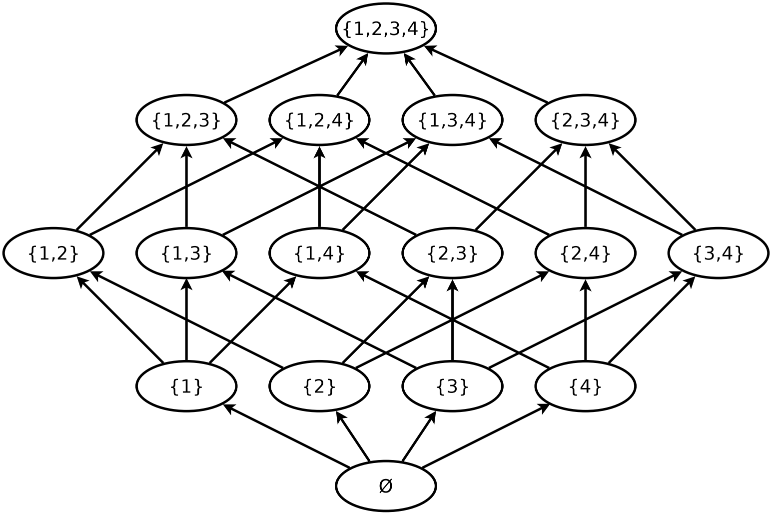
Boolean decision diagrams

In particular, we can formulate it as a path search problem.

- A state is a set of variables whose position in the order has been fixed.
- Each transition fixes the position of a variable.
- A path of length k defines the first k variables in the ordering.

Variable orderings

A permutation corresponds to a path in the subset lattice.



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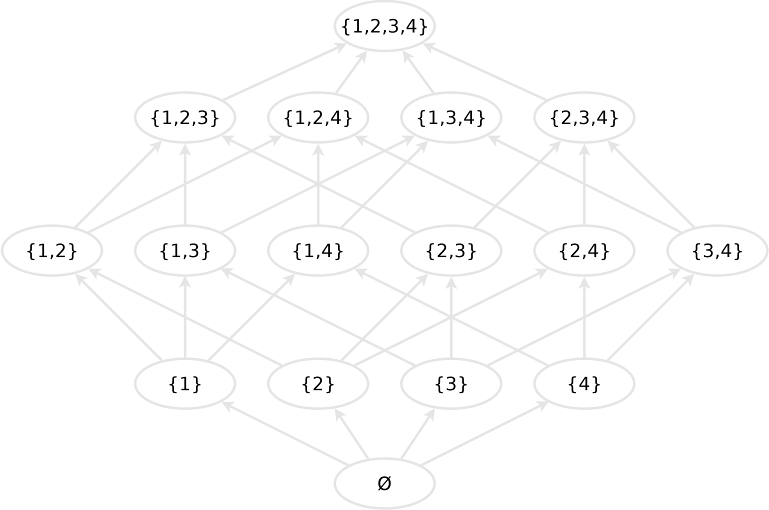

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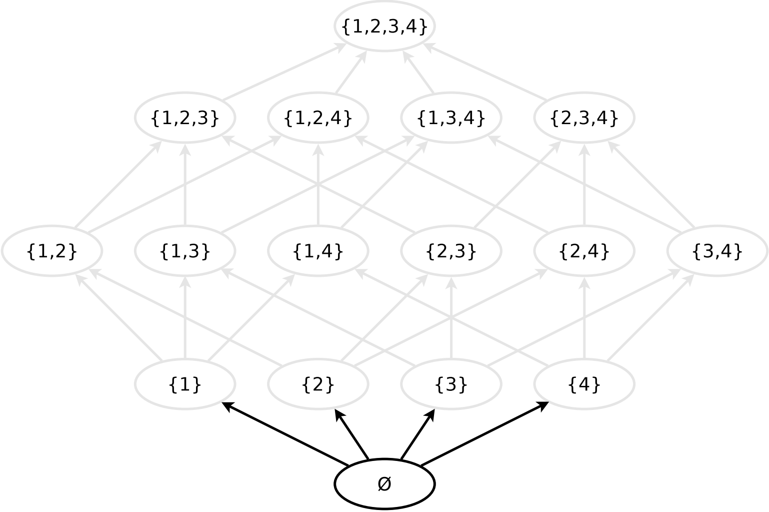

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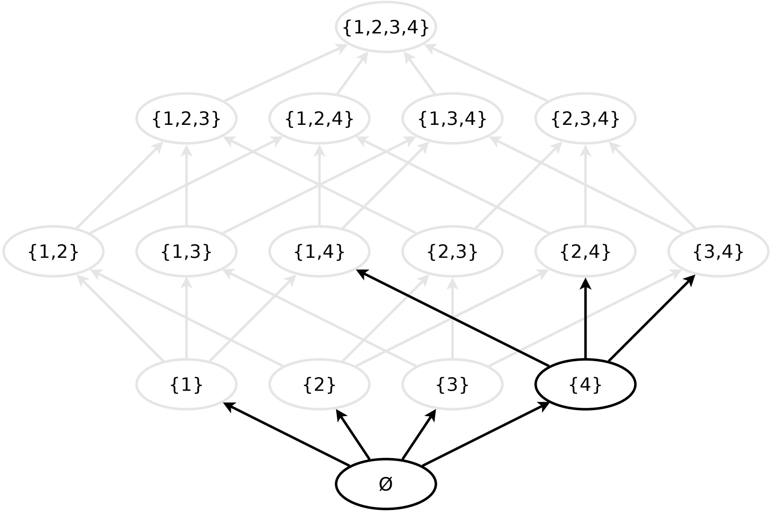

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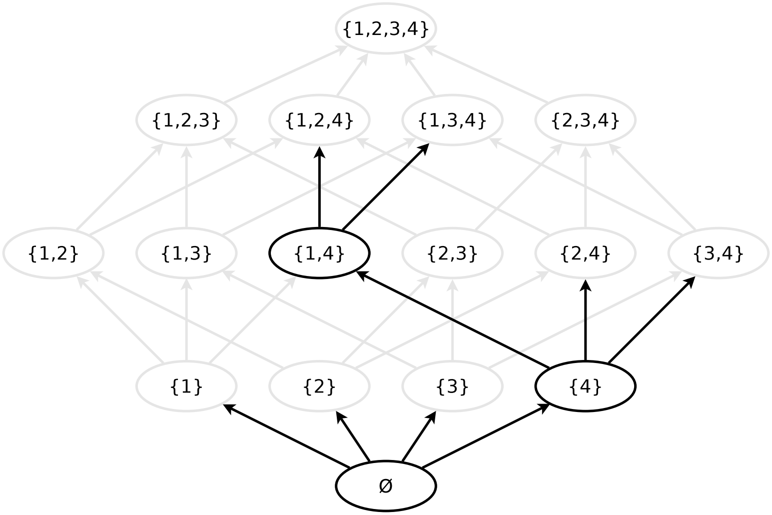

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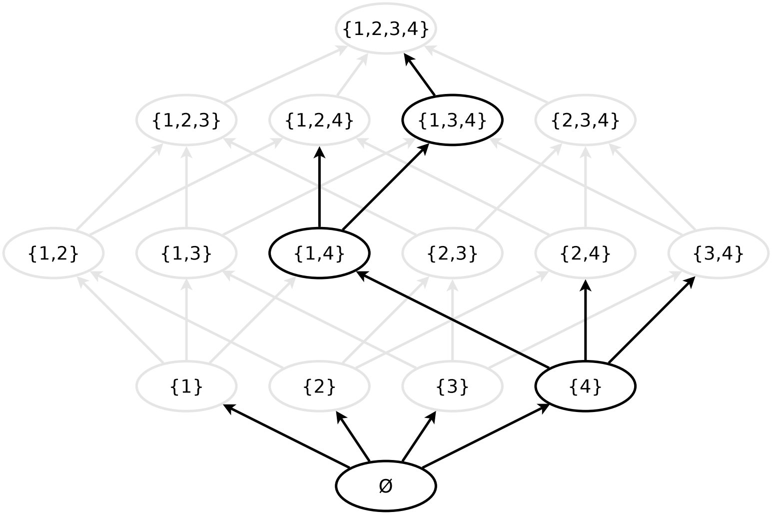

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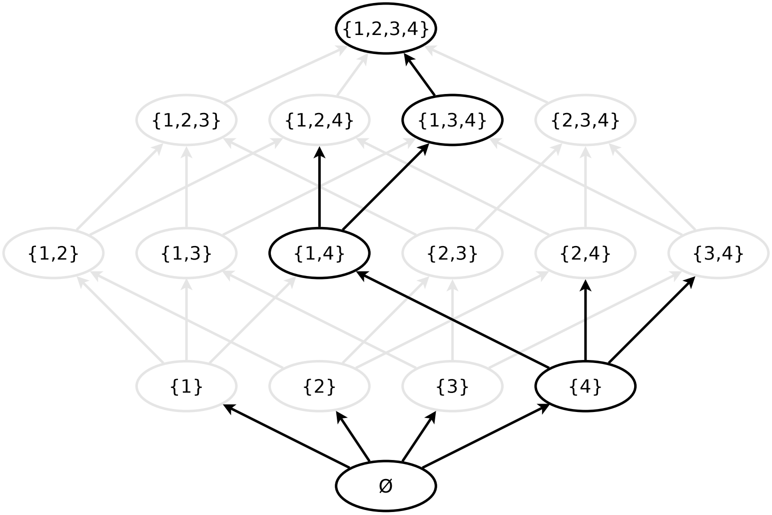
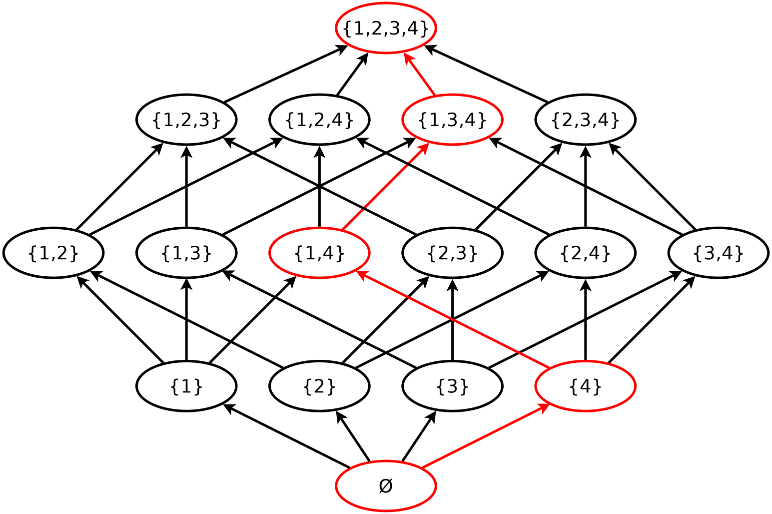


Variable orderings

A permutation corresponds to a path in the subset lattice: 4,1,3,2



Variable orderings

Finding an optimal path is equivalent to finding an optimal BDD.

> g: For a path of length k, the size of the first k levels of the BDD.
> h: The number of cofactors: a lower bound on the size of the BDD.

Weighted A* used for approximate BDD mimimization.

Experiments

Experimental results.

Questions?