1. Divide $X^4 + X^3 + X + 1$ by $X^2 + X + 1$, when the coefficients are modulo 2 integers.

2. Divide $3X^5 + 4X^4 + 2X^3 + 2X^2 + 2X + 4$ by $4X^3 + 1$ when the coefficients are modulo 5 integers.

3. Construct the addition table for the finite field $GF(5^2)$.

4. Construct the multiplication table for $GF(5^2)$, when the irreducible polynomial is $X^2 + X + 1$.

5. Consider the finite field $GF(2^8)$ used in the AES encryption. It was defined with the help of the irreducible polynomial $M(X) = X^8 + X^4 + X^3 + X + 1$.

   Let $P(X) = X^5 + X^4 + X + 1$ and $Q(X) = X^7 + X^3 + X^2$.

   a) Show how these polynomials can be represented in a byte.

   b) Calculate $X \cdot P(X) \mod M(X)$ using bitwise operations and write the result in a normal symbolic form.

   c) Calculate $X \cdot Q(X) \mod M(X)$ using a) the normal highschool algebra and b) the bitwise operations.

6. Suppose that in a AES state the elements $(0, 0)$ and $(0, 1)$ contain 1 and all the other elements contain 0. What is the state after the MixColumns operation?