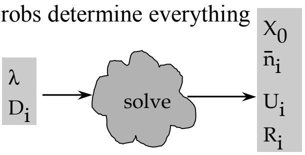
Lecture 10
Open Models

Open Models

Mixed Models

Open Queuing Networks

- Easy because system throughput is already known (same as arrival rate!)
  - Forced Flow Law (FFL) and branching probs determine everything



λ
D_i

X_0
$\bar{n}_i$
U_i
R_i

not ir!

Arrival Thm: $\bar{n}_i^A(\lambda) = \bar{n}_i(\lambda)$ $\bar{n}_{ir}^A(\lambda) = \bar{n}_i(\lambda)$

One Class Open Network Solution (3)

$$X_0 = \lambda \qquad X_i = X_0 V_i = \lambda V_i$$
$$U_i = X_i S_i = \lambda V_i S_i = \lambda D_i = X_0 D_i$$

Need S_i or D_i?

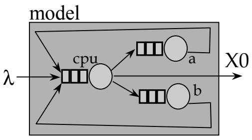
$$\left.\begin{aligned} \bar{n}_i = X_i R_i = X_i S_i\left[1+\bar{n}_i\right] = U_i\left[1+\bar{n}_i\right] \Rightarrow \bar{n}_i = \frac{U_i}{1-U_i} \\ R_i = S_i\left[1+\bar{n}_i\right] = \frac{S_i}{1-U_i} \end{aligned}\right\} \text{(queue dev)}$$

$$\bar{n}_i = X_i R_i = X_i S_i = U_i \quad R_i = S_i \qquad \text{(delay dev)}$$

$$R_i^{'} = V_i R_i \qquad R = \sum R_i^{'} = \sum V_i R_i \qquad \bar{n} = \sum \bar{n}_i$$

Must have $U_i \le 1.0$ for all i. Why? What if $U_i > 1.0$?

Open Network



measurements

T = 3600 s
Busy: B_{cpu} =1728
B_a = 1512 B_b = 2592
Arrivals: A =10800
A_a = 75600 A_b = 86400

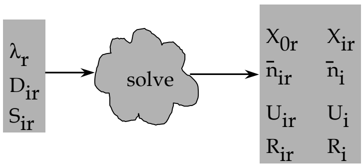
visit ratios, service times, solution

X_0 = 10800/3600 = 3
V_{cpu} = 1+7+8 = 16 V_a = 75600/10800 = 7 V_b = 86400/10800 = 8
D_{cpu} = 1728/10800 = 0.16 D_a = 1512/10800 = 0.14 s D_b = 2592/10800 =0.24 s
S_{cpu} = 0.16/16 = 0.01 s S_a = 0.14/7 = 0.02 s S_b = 0.24/8 = 0.03 s

U_{cpu} = X_0 D_{cpu} = 0.48 U_a = 3 x 0.14 = 0.42 U_b = 0.72
R_{cpu} = S_{cpu}/(1-U_{cpu}) = 0.0192 R_a = 0.0345 R_b = 0.03/0.28=0.107
R = $\Sigma V_i R_i$ = 1.406 s
$\bar{N}_{cpu}$ = U_{cpu}/(1-U_{cpu}) = 0.923 $\bar{N}_a$ = 0.42/0.58=0.72 $\bar{N}_b$ = 0.72/28 = 2.571
$\bar{N}$ = $\Sigma \bar{N}_i$ = 4.214

Multiple Class Open Network

- Still "easy" though more complex than single class case



λ_r
D_{ir}
S_{ir}

X_{0r} X_{ir}
$\bar{n}_{ir}$ $\bar{n}_i$
U_{ir} U_i
R_{ir} R_i

$$\bar{n}_{ir} = \frac{U_{ir}}{1-U_i} = \frac{\lambda_r D_{ir}}{1-U_i}$$ etc etc

Multiple Class Open Network

$$X_{0r} = \lambda_r \qquad X_{ir} = X_0 V_{ir} = \lambda_r V_{ir}$$

$$U_{ir} \overset{Little}{=} X_{0r} V_{ir} S_{ir} = \lambda_r D_{ir} = X_{0r} D_{ir} \qquad U_i = \sum U_{ir} \le 1.0 \text{ must have!}$$

$$R_{ir}^{'} = D_{ir}\left[1+\bar{n}_{ir}^{A}(\lambda)\right] = D_{ir}\left[1+\bar{n}_i\right]$$

Fig. 6.7
[Men 94]

$$\bar{n}_{ir} \overset{Little}{=} \lambda_r V_{ir} R_{ir} = \lambda_r R_{ir}^{'} = \lambda_r D_{ir}\left[1+\bar{n}_i\right] = U_{ir}\left[1+\bar{n}_i\right]$$

$$\therefore 1+\bar{n}_i = \bar{n}_{ir}/U_{ir} = \bar{n}_{is}/U_{is} \quad \forall s$$

$$\bar{n}_{ir} = U_{ir}\left[1+\sum_s \bar{n}_{is}\right] = U_{ir} + \sum_s U_{ir}\bar{n}_{is} = U_{ir} + \sum_s \frac{\bar{n}_{ir}}{\cancel{\bar{n}_{is}}} U_{is}\cancel{\bar{n}_{is}} = U_{ir} + \bar{n}_{ir} U_i$$

$$\therefore \bar{n}_{ir} = \frac{U_{ir}}{1-U_i} = \frac{\lambda_r D_{ir}}{1-U_i} \text{ and } R_{ir}^{'} = \frac{\bar{n}_{ir}}{\lambda_r} = \frac{D_{ir}}{1-U_i} \qquad \text{(queue dev)}$$

Need S_{ir}!

$$R_{ir} = \frac{S_{ir}}{1-U_i} \qquad R_r = \sum_i R_{ir}^{'} \qquad \bar{n}_i = \sum \bar{n}_{ir} \qquad \bar{n} = \sum \bar{n}_i$$

Multiple Class Open Network Solution (8)

$X_{0r} = \lambda_r \qquad X_{ir} = X_0 V_{ir} = \lambda_r V_{ir}$

$U_{ir} \overset{Little}{=} X_{0r} V_{ir} S_{ir} = \lambda_r D_{ir} = X_{0r} D_{ir} \qquad U_i = \sum U_{ir} \le 1.0$ must have!

$R'_{ir} = D_{ir}\left[1 + \bar{n}^{A}_{ir}(\lambda)\right] = D_{ir}\left[1 + \bar{n}_i\right]$

Fig. 6.7
[Men 94]

$\bar{n}_{ir} \overset{Little}{=} \lambda_r V_{ir} R_{ir} = \lambda_r R'_{ir} = \lambda_r D_{ir}\left[1 + \bar{n}_i\right] = U_{ir}\left[1 + \bar{n}_i\right]$

$\therefore\ 1 + \bar{n}_i = \bar{n}_{ir} / U_{ir} = \bar{n}_{is} / U_{is} \quad \forall s$

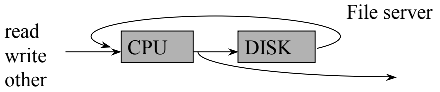
$\bar{n}_{ir} = U_{ir}\left[1 + \sum_s \bar{n}_{is}\right] = U_{ir} + \sum_s U_{ir} \bar{n}_{is} = U_{ir} + \sum_s \frac{\bar{n}_{ir}}{\cancel{\bar{n}_{is}}} U_{is} \cancel{\bar{n}_{is}} = U_{ir} + \bar{n}_{ir} U_i$

$\therefore\ \bar{n}_{ir} = \frac{U_{ir}}{1 - U_i} = \frac{\lambda_r D_{ir}}{1 - U_i}$ and $R'_{ir} = \frac{\bar{n}_{ir}}{\lambda_r} = \frac{D_{ir}}{1 - U_i}$ (queue dev)

$R_{ir} = \frac{S_{ir}}{1 - U_i} \qquad R_r = \sum_i R'_{ir} \qquad \bar{n}_i = \sum \bar{n}_{ir} \qquad \bar{n} = \sum \bar{n}_i$

Need S_{ir}!

Open Model Example



measurement

T=3600 sec 18000 file reads, 7200 writes, 3600 other
5 /sec 2 / sec 1 / sec
CPU util 32% (9% read, 18% write, 5% other)
disk util 48% (20% read, 20% write, 8% other)

$D_{disk,read} = U_{disk,read} / X_{0,read} = 0.2/5 = 0.040$, $D_{cpu,read} = 0.018$
$D_{disk,write} = 0.1$, $D_{cpu,write} = 0.090$
$D_{disk,other} = 0.080$, $D_{cpu,write} = 0.050$

Open Model Example (contd)

$D_{disk,read} = U_{disk,read} / X_{0,read} = 0.2/5 = 0.040$, $D_{cpu,read} = 0.018$
$D_{disk,write} = 0.1$, $D_{cpu,write} = 0.090$
$D_{disk,other} = 0.080$, $D_{cpu,write} = 0.050$

HW specs → $S_{disk} = 0.020$

central server model

$V_{disk,read} = 2 \quad V_{disk,write} = 5 \quad V_{disk,other} = 4$

$V_{proc,read} = 1 + V_{disk,read} = 3 \quad V_{proc,write} = 6 \quad V_{proc,other} = 5$

$S_{proc,read} = 0.006 \quad S_{proc,write} = 0.015 \quad S_{proc,other} = 0.100$

(Tbl 6.8)

Open Model Example (contd)

Try first simple single class model?

weight = system arrival rate

$D_{cpu} = (0.018 * 5 + 0.090 * 2 + 0.050 * 1) / 8 = 0.040$
$D_{disk} = (0.04 * 5 + 0.10 * 2 + 0.08 * 1) / 8 = 0.06$
$\lambda = 8$ (jobs per sec)

$U_{cpu} = \lambda D_{cpu} = 8 * 0.040 = 0.320 \qquad U_{disk} = 0.480$

$R'_{cpu} = D_{cpu}/(1 - U_{cpu}) = 0.040/0.680 = 0.059$
$R'_{disk} = 0.060/0.520 = 0.115$
$R = 0.174$

Open Model Example (contd)

Modification A. What if twice as many workstations? λ= 16

$U_{cpu} = \lambda D_{cpu} = 16 * 0.040 = 0.640 \qquad U_{disk} =$ **0.960 (!!)**

$R'_{cpu} = D_{cpu}/(1 - U_{cpu}) = 0.040/0.360 = 0.111$
$R'_{disk} = 0.060/0.04 = 1.5$
$R = 1.611$

Not good. How to get R down?

Open Model Example (contd)

Modification B. Also server cache: 70% hit ratio for reads

$V_{disk,read}$ goes down 70%, $D_{disk,read} = 0.3 * 0.040 = 0.012$

D_{ir}	Read	Write	Other
CPU	0.018	0.090	0.050
DISK	0.012	0.100	0.080
λ_r	10	4	2

$U_{ir} = \lambda_r D_{ir} \qquad U_i = \sum U_{ir}$
$R'_{ir} = D_{ir}/(1 - U_i)$
$n_{ir} = U_{ir}/(1 - U_i)$
$n_i = \sum n_{ir}$
$n = \sum n_i = 3.925$

see next slide on server cache calculations!

R= (0.0875, 0.5625, 0.389) OK

Open Model Example (contd)

$X_o = (10\ \ 4\ \ 2) = \lambda$

10*0.018

4*0.09

$U_{ir} = \lambda_r D_{ir}$	Read	Write	Other	U_i
CPU	0.180	0.36	0.10	0.64
Disk	0.120	0.400	0.16	0.68

$R'_{ir} = D_{ir}/(1-U_i)$	Read	Write	Other
CPU	0.050	0.25	0.139
Disk	0.375	0.3125	0.25
R_r	0.425	0.5625	0.389

0.018/0.36

$N_{ir} = U_{ir}/(1-U_i)$	Read	Write	Other	N_i
CPU	0.50	1.0	0.278	1.778
Disk	0.375	1.25	0.5	2.125

0.18/0.36

sum=3.925

Open Model Example (contd)

Modification C. Client Cache (write through)

Save 70% of reads: λ_r = 30% 10 = 3

Modification D. Disk Upgrade:
Another similar disk

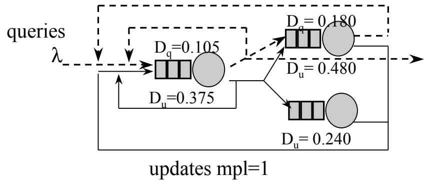
$D_{disk1,r} = D_{disk2,r} = D_{disk,r}/2$

Tbl 6.9

weighted average response time: Tbl 6.10

Mixed Models

- Open and closed job classes



- Load independent servers

Mixed Models Solution

- 1. Solve for open job classes

λ_r, D_{ir} → open → $X_{0r}\ X_{ir}$, $U_{ir}\ U_i^{open}$

NOT: $R_{ir}\ R'_{ir}\ n_{ir}$

- 2. Slow down for closed classes

$D_{ir}^{new} = D_{ir}^{new}/(1-U_i^{open})$

- 3. Solve for closed classes with MVA

n_r, D_{ir}^{new} → mva → $X_{0r}\ X_{ir}\ R'_{ir}\ R_r$, $n_{ir}\ n_i^{closed}$

- 4. Solve R'_{ir}, R_r, n_{ir} for open classes

open → $R'_{ir} = D_{ir}(1+n_i^{closed})/(1-U_i^{open})$
$n_{ir} = X_{0,r} R'_{ir} = \lambda_r R'_{ir}$

Step 4?

- 4. Solve R'_{ir}, R_r, n_{ir} for open classes

why only “closed”? why only “open”?

open → $R'_{ir} = D_{ir}(1+n_i^{closed})/(1-U_i^{open})$
$n_{ir} = X_{0,r} R'_{ir} = \lambda_r R'_{ir}$

plain open model: $R'_{ir} = D_{ir}/(1-U_i)$

Mixed Model Example (2)

Measurement data: Tbl 6.11

1. $U_{cpu,q} = \lambda D_{ir} = 4.09 * 0.105 = 0.4295$
   $U_{D1,q} = 4.09 * 0.180 = 0.7362 \quad U_{D2,q} = 4.09 * 0 = 0$

2. $D_{cpu,u} = 0.375/(1-0.4295) = 0.375/0.5705 = 0.657$
   $D_{D1,u} = 0.480/0.2638 = 1.820 \quad D_{D1,u} = 0.240/1 = 0.240$

3. $R_u = \sum D_{ir} = 2.717$
   $X_{0,u} = N/R = 0.368$
   $n_{cpu,u} = X_{0,u} R'_{cpu,u} = X_{0,u} D_{cpu,u} = 0.368*0.657 = 0.242$
   $n_{D1,u} = 0.368*1.820 = 0.670$
   $n_{D2,u} = 0.368*0.240 = 0.088$
   $U_{cpu,u} = X_{0,u} D_{cpu,u} = n_{cpu,u}$

Mixed Model Example (contd)

4. $R'_{cpu,q} = D_{cpu,q} (1+n_{cpu}^{closed}) / (1-U_{cpu}^{open})$
   $= 0.105 (1+0.242)/(1-0.4295) = 0.229$
   $R'_{D1,q} = 0.180 (1+0.670)/(1-0.7362) = 1.140$
   $R'_{D2,q} = 0$
   $R_q = \sum R'_{iq} = 1.369$
   $n_{cpu,q} = X_{0,q} R'_{cpu,q} = 4.09 * 0.229 = 0.9366$
   $n_{D1,q} = 4.09 * 1.140 = 4.6626 \quad n_{D2,q} = 0$