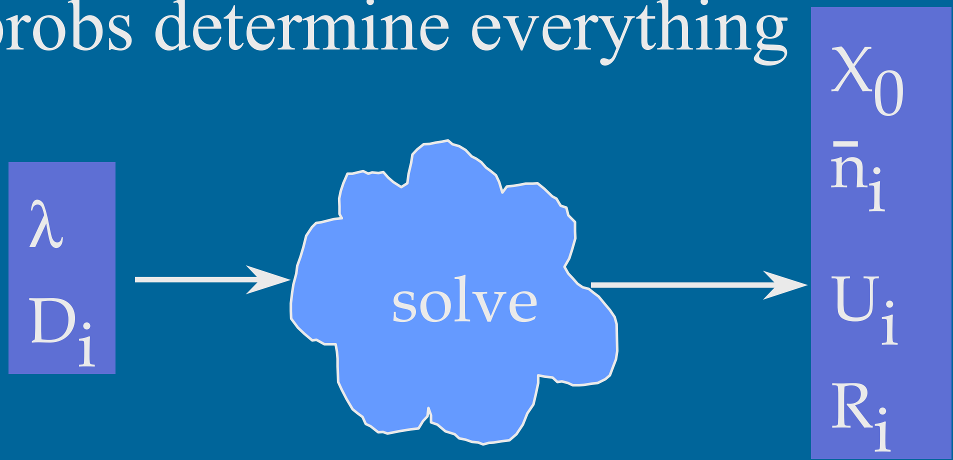
Lecture 10
Open Models

Open Models

Mixed Models

Open Queuing Networks

- Easy because system throughput is already known (same as arrival rate!)
  - Forced Flow Law (FFL) and branching probs determine everything



not ir!

Arrival Thm: $\bar{n}_i^A(\lambda) = \bar{n}_i(\lambda)$ $\bar{n}_{ir}^A(\lambda) = \bar{n}_i(\lambda)$

One Class Open Network Solution (3)

$$X_0 = \lambda \qquad X_i = X_0 V_i = \lambda V_i$$

$$U_i = X_i S_i = \lambda V_i S_i = \lambda D_i = X_0 D_i$$

Need S_i or D_i?

$$\bar{n}_i = X_i R_i = X_i S_i [1 + \bar{n}_i] = U_i [1 + \bar{n}_i] \Rightarrow \bar{n}_i = \frac{U_i}{1 - U_i}$$

$$R_i = S_i [1 + \bar{n}_i] = \frac{S_i}{1 - U_i}$$

(queue dev)

$$\bar{n}_i = X_i R_i = X_i S_i = U_i \qquad R_i = S_i$$

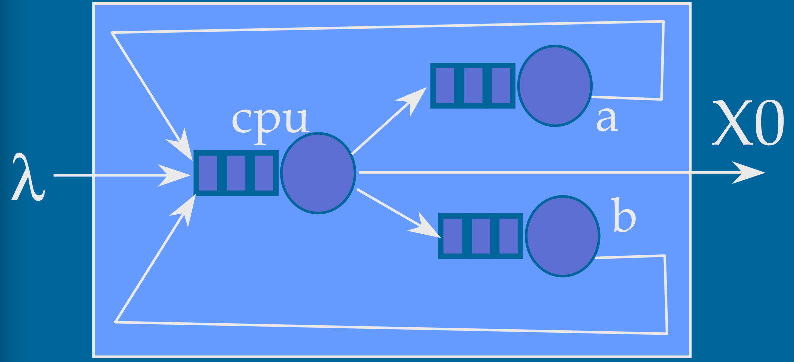
(delay dev)

$$R_i' = V_i R_i \qquad R = \sum R_i' = \sum V_i R_i \qquad \bar{n} = \sum \bar{n}_i$$

Must have $U_i \le 1.0$ for all i. Why? What if $U_i > 1.0$?

Open Network

model



measurements

T = 3600 s
Busy: B_{cpu} =1728
B_a = 1512 B_b = 2592
Arrivals: A =10800
A_a = 75600 A_b = 86400

visit ratios, service times, solution

$X_0 = 10800/3600 = 3$

$V_{cpu} = 1+7+8 = 16$	$V_a = 75600/10800 = 7$	$V_b = 86400/10800 = 8$
$D_{cpu} = 1728/10800 = 0.16$	$D_a = 1512/10800 = 0.14$ s	$D_b = 2592/10800 = 0.24$ s
$S_{cpu} = 0.16/16 = 0.01$ s	$S_a = 0.14/7 = 0.02$ s	$S_b = 0.24/8 = 0.03$ s

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$U_{cpu} = X_0 D_{cpu} = 0.48$	$U_a = 3 \times 0.14 = 0.42$	$U_b = 0.72$
$R_{cpu} = S_{cpu}/(1-U_{cpu}) = 0.0192$	$R_a = 0.0345$	$R_b = 0.03/0.28 = 0.107$

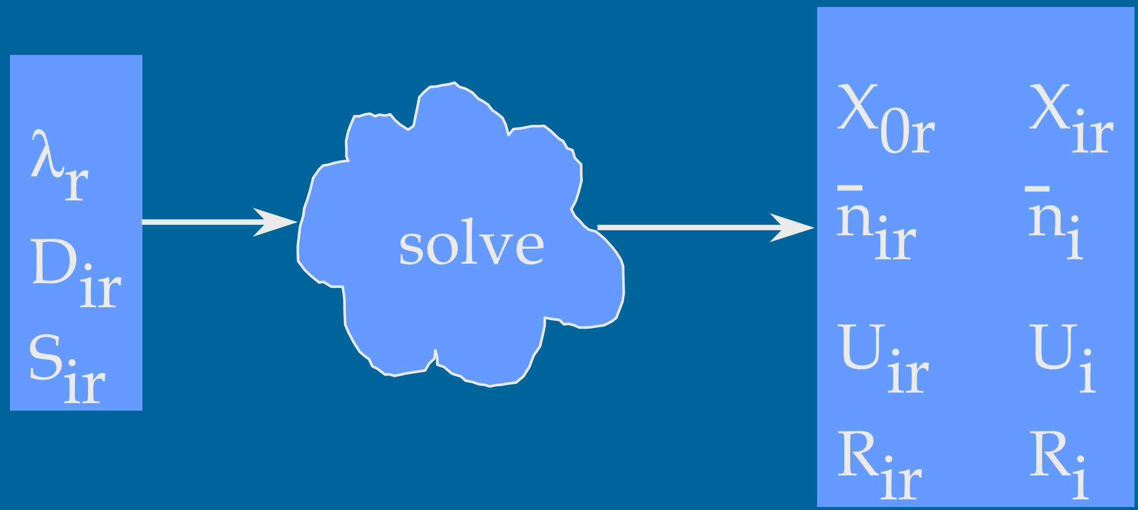
$R = \Sigma V_i R_i = 1.406$ s

$\bar{N}_{cpu} = U_{cpu}/(1-U_{cpu}) = 0.923$ $\bar{N}_a = 0.42/0.58 = 0.72$ $\bar{N}_b = 0.72/28 = 2.571$

$\bar{N} = \Sigma \bar{N}_i = 4.214$

Multiple Class Open Network

- Still "easy" though more complex than single class case



$$\bar{n}_{ir} = \frac{U_{ir}}{1-U_i} = \frac{\lambda_r D_{ir}}{1-U_i}$$ etc etc

Multiple Class Open Network

$$X_{0r} = \lambda_r \qquad X_{ir} = X_0 V_{ir} = \lambda_r V_{ir}$$

$$U_{ir} \overset{Little}{=} X_{0r} V_{ir} S_{ir} = \lambda_r D_{ir} = X_{0r} D_{ir} \qquad U_i = \sum U_{ir} \le 1.0 \text{ must have!}$$

$$R'_{ir} = D_{ir}\left[1 + \bar{n}^{A}_{ir}(\lambda)\right] = D_{ir}\left[1 + \bar{n}_i\right]$$

$$\bar{n}_{ir} \overset{Little}{=} \lambda_r V_{ir} R_{ir} = \lambda_r R'_{ir} = \lambda_r D_{ir}\left[1 + \bar{n}_i\right] = U_{ir}\left[1 + \bar{n}_i\right]$$

Fig. 6.7
[Men 94]

$$\therefore 1 + \bar{n}_i = \bar{n}_{ir} / U_{ir} = \bar{n}_{is} / U_{is} \quad \forall s$$

$$\bar{n}_{ir} = U_{ir}\left[1 + \sum_s \bar{n}_{is}\right] = U_{ir} + \sum_s U_{ir} \bar{n}_{is} = U_{ir} + \sum_s \frac{\bar{n}_{ir}}{\bar{n}_{is}} U_{is} \bar{n}_{is} = U_{ir} + \bar{n}_{ir} U_i$$

$$\therefore \bar{n}_{ir} = \frac{U_{ir}}{1 - U_i} = \frac{\lambda_r D_{ir}}{1 - U_i} \quad \text{and} \quad R'_{ir} = \frac{\bar{n}_{ir}}{\lambda_r} = \frac{D_{ir}}{1 - U_i} \qquad \text{(queue dev)}$$

Need S_{ir}!

$$R_{ir} = \frac{S_{ir}}{1 - U_i} \qquad R_r = \sum_i R'_{ir} \qquad \bar{n}_i = \sum \bar{n}_{ir} \qquad \bar{n} = \sum \bar{n}_i$$

Multiple Class Open Network Solution (8)

$$X_{0r} = \lambda_r \qquad X_{ir} = X_0 V_{ir} = \lambda_r V_{ir}$$

$$U_{ir} \overset{Little}{=} X_{0r} V_{ir} S_{ir} = \lambda_r D_{ir} = X_{0r} D_{ir} \qquad U_i = \sum U_{ir} \le 1.0 \text{ must have!}$$

$$R'_{ir} = D_{ir}\left[1 + \bar{n}^A_{ir}(\lambda)\right] = D_{ir}\left[1 + \bar{n}_i\right]$$

$$\bar{n}_{ir} \overset{Little}{=} \lambda_r V_{ir} R_{ir} = \lambda_r R'_{ir} = \lambda_r D_{ir}\left[1 + \bar{n}_i\right] = U_{ir}\left[1 + \bar{n}_i\right]$$

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$$\therefore 1 + \bar{n}_i = \bar{n}_{ir} / U_{ir} = \bar{n}_{is} / U_{is} \quad \forall s$$

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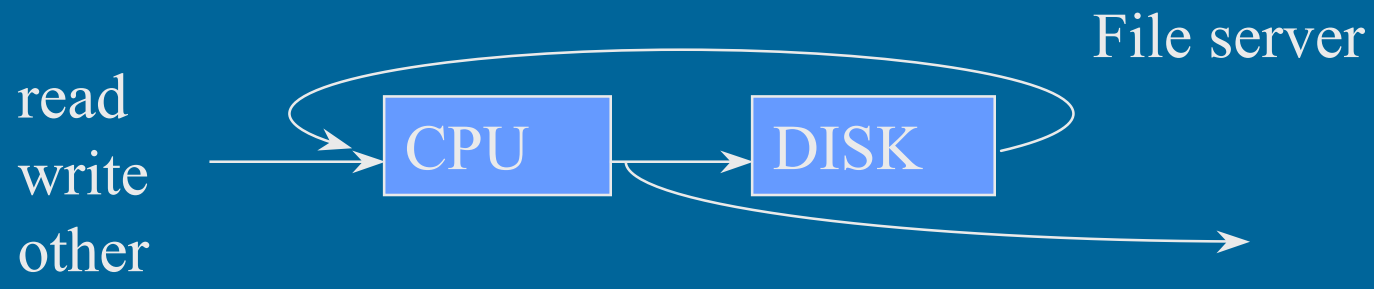
$$\therefore \bar{n}_{ir} = \frac{U_{ir}}{1-U_i} = \frac{\lambda_r D_{ir}}{1-U_i} \text{ and } R'_{ir} = \frac{\bar{n}_{ir}}{\lambda_r} = \frac{D_{ir}}{1-U_i} \qquad \text{(queue dev)}$$

$$R_{ir} = \frac{S_{ir}}{1-U_i} \qquad R_r = \sum_i R'_{ir} \qquad \bar{n}_i = \sum \bar{n}_{ir} \qquad \bar{n} = \sum \bar{n}_i$$

Need S_{ir}!

Open Model Example



measurement

T=3600 sec	18000 file reads,	7200 writes,	3600 other
	5 /sec	2 / sec	1 / sec

CPU util 32% (9% read, 18% write, 5% other)

disk util 48% (20% read, 20% write, 8% other)

$D_{disk,read} = U_{disk,read} / X_{0,read} = 0.2/5 = 0.040$, $D_{cpu,read} = 0.018$

$D_{disk,write} = 0.1$, $D_{cpu,write} = 0.090$

$D_{disk,other} = 0.080$, $D_{cpu,write} = 0.050$

Open Model Example (contd)

$D_{disk,read} = U_{disk,read} / X_{0,read} = 0.2/5 = 0.040$, $D_{cpu,read} = 0.018$

$D_{disk,write} = 0.1$, $D_{cpu,write} = 0.090$

$D_{disk,other} = 0.080$, $D_{cpu,write} = 0.050$

HW specs → $S_{disk} = 0.020$

$V_{disk,read} = 2$ $V_{disk,write} = 5$ $V_{disk,other} = 4$

central server model

$V_{proc,read} = 1 + V_{disk,read} = 3$ $V_{proc,write} = 6$ $V_{proc,other} = 5$

$S_{proc,read} = 0.006$ $S_{proc,write} = 0.015$ $S_{proc,other} = 0.100$

(Tbl 6.8)

Open Model Example (contd)

Try first simple single class model?

weight = system arrival rate

D_{cpu} = (0.018 * 5 + 0.090 * 2 + 0.050 * 1) / 8 = 0.040
D_{disk} = (0.04 * 5 + 0.10 * 2 + 0.08 *1) / 8 = 0.06
λ = 8 (jobs per sec)

$U_{cpu} = \lambda D_{cpu} = 8 * 0.040 = 0.320$ $U_{disk} = 0.480$

$R'_{cpu} = D_{cpu}/(1- U_{cpu})=0.040/0.680 = 0.059$
$R'_{disk} = 0.060/0.520 = 0.115$
$R = 0.174$

Open Model Example (contd)

Modification A. What if twice as many workstations? $\lambda = 16$

$U_{cpu} = \lambda\, D_{cpu} = 16 * 0.040 = 0.640 \qquad U_{disk} = 0.960$ (!!)

$R'_{cpu} = D_{cpu}/(1 - U_{cpu}) = 0.040/0.360 = 0.111$
$R'_{disk} = 0.060/0.04 = 1.5$
$R = 1.611$

Not good. How to get R down?

Open Model Example (contd)

Modification B. Also server cache: 70% hit ratio for reads

$V_{disk,read}$ goes down 70%, $D_{disk,read} = 0.3 * 0.040 = 0.012$

D_{ir}	Read	Write	Other
CPU	0.018	0.090	0.050
DISK	0.012	0.100	0.080
λ_r	10	4	2

$U_{ir} = \lambda_r D_{ir} \quad U_i = \sum U_{ir}$

$R'_{ir} = D_{ir}/(1-U_i)$

$n_{ir} = U_{ir}/(1-U_i)$

$n_i = \sum n_{ir}$

$n = \sum n_i = 3.925$

see next slide on server cache calculations!

R= (0.0875, 0.5625, 0.389) OK

Open Model Example (contd)

$X_o = (10 \;\; 4 \;\; 2)$
$= \lambda$

10*0.018   4*0.09

$U_{ir} = \lambda_r D_{ir}$	Read	Write	Other	U_i
CPU	0.180	0.36	0.10	0.64
Disk	0.120	0.400	0.16	0.68

$R'_{ir} = D_{ir}/(1-U_i)$	Read	Write	Other
CPU	0.050	0.25	0.139
Disk	0.375	0.3125	0.25
R_r	0.425	0.5625	0.389

0.018/0.36

$N_{ir} = U_{ir}/(1-U_i)$	Read	Write	Other	N_i
CPU	0.50	1.0	0.278	1.778
Disk	0.375	1.25	0.5	2.125

0.18/0.36

sum=3.925

Open Model Example (contd)

Modification C. Client Cache (write through)

Save 70% of reads: $\lambda_r = 30\%\ 10 = 3$

Modification D. Disk Upgrade:
Another similar disk

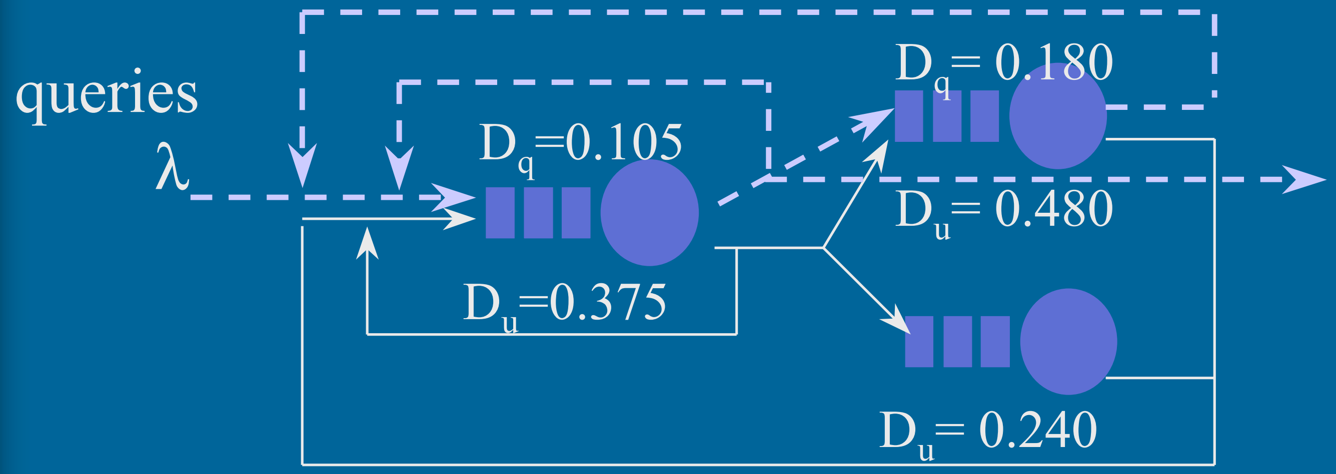
$$D_{disk1,r} = D_{disk2,r} = D_{disk,r}/2$$

Tbl 6.9

weighted average response time: Tbl 6.10

Mixed Models

- Open and closed job classes



- Load independent servers

Mixed Models Solution

- 1. Solve for open job classes

$\lambda_r, D_{ir} \rightarrow$ open $\rightarrow X_{0r}\ X_{ir}\ U_{ir}\ U_i^{open}$

NOT: $R_{ir}\ R'_{ir}\ n_{ir}$

- 2. Slow down for closed classes

$$D_{ir}^{new} = D_{ir}^{new} / (1 - U_i^{open})$$

- 3. Solve for closed classes with MVA

$n_r, D_{ir}^{new} \rightarrow$ mva $\rightarrow X_{0r}\ X_{ir}\ R'_{ir}\ R_r\ n_{ir}\ n_i^{closed}$

- 4. Solve R'_{ir}, R_r, n_{ir} for open classes

open $\rightarrow$

$$R'_{ir} = D_{ir} (1 + n_i^{closed}) / (1 - U_i^{open})$$

$$n_{ir} = X_{0,r} R'_{ir} = \lambda_r R'_{ir}$$

Step 4?

- 4. Solve R'_{ir}, R_r, n_{ir} for open classes

why only "closed"? why only "open"?

open

$$R'_{ir} = D_{ir}\,(1+n_i^{closed}) \,/\, (1-U_i^{open})$$

$$n_{ir} = X_{0,r}\,R'_{ir} = \lambda_r\,R'_{ir}$$

plain open model: $R'_{ir} = D_{ir}/(1-U_i)$

Mixed Model Example (2)

Measurement data: Tbl 6.11

1. $U_{cpu,q} = \lambda D_{ir} = 4.09 * 0.105 = 0.4295$
   $U_{D1,q} = 4.09 * 0.180 = 0.7362 \qquad U_{D2,q} = 4.09 * 0 = 0$

2. $D_{cpu,u} = 0.375/(1-0.4295) = 0.375/0.5705 = 0.657$
   $D_{D1,u} = 0.480/0.2638 = 1.820 \qquad D_{D1,u} = 0.240/1 = 0.240$

3. $R_u = \sum D_{ir} = 2.717$
   $X_{0,u} = N/R = 0.368$
   $n_{cpu,u} = X_{0,u} R'_{cpu,u} = X_{0,u} D_{cpu,u} = 0.368*0.657 = 0.242$
   $n_{D1,u} = 0.368*1.820 = 0.670$
   $n_{D2,u} = 0.368*0.240 = 0.088$
   $U_{cpu,u} = X_{0,u} D_{cpu,u} = n_{cpu,u}$

Mixed Model Example (contd)

4.

$R'_{cpu,q} = D_{cpu,q} (1+n_{cpu}^{closed}) / (1-U_{cpu}^{open})$

$= 0.105 (1+0.242)/(1-0.4295) = 0.229$

$R'_{D1,q} = 0.180 (1+0.670)/(1-0.7362) = 1.140$

$R'_{D2,q} = 0$

$R_q = \sum R'_{iq} = 1.369$

$n_{cpu,q} = X_{0,q} R'_{cpu,q} = 4.09 * 0.229 = 0.9366$

$n_{D1,q} = 4.09 * 1.140 = 4.6626$ $\quad n_{D2,q} = 0$