Lecture 8
MVA, Hierarchical Models
Solution Packages

MVA

Hierarchical Models

PMVA

Mean Value Analysis (MVA)

- A new way to compute the same results as with convolution
- Can compute results without state space enumeration (have class population enumeration)
- Based on Arrival Theorem:
  - With N jobs in the system, a job arriving to some server i sees the server as in equilibrium with one job (himself!) removed from the system

$A_i(N) = \bar{n}_i(N - \mathbf{1})$

Arrival Theorem Example

- 2 jobs in closed network: CPU & 2 disks
- Fig. 5.5
- One job (J) in transition (in a "cloud")
- Look at steady state job distribution for 1 job in the network
  - (e.g., 0.4 at CPU, .3 at both disks)
- Job J sees in average 0.4 jobs at CPU when it arrives there

N=0 → N=1 → N=2 → N=3 → N=4 → …

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Single Class MVA Algorithm

starting point: $A_i(N) = \bar{n}_i(N-\mathbf{1})$ N=1 $\bar{n}_i(\mathbf{0}) = 0$

residence time

$$R_i'(N) = \begin{cases} D_i[1 + A_i(N)] \overset{\text{arrival thm}}{=} D_i[1 + \bar{n}_i(N-1)] & \textit{queueing device} \\ D_i & \textit{delay device} \end{cases}$$

$$X_0(N) \underset{system}{\overset{Little}{=}} \frac{N}{Z + \sum_k R_k'(N)}$$ (response time law)

N+1

$$\bar{n}_i(N) \underset{device}{\overset{Little}{=}} X_i(N) R_i(N) = \underbrace{X_0(N) V_i}_{X_i(N)} \overbrace{R_i(N)}^{R_i'(N)}$$

$$= X_0(N) R_i'(N)$$

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MVA Example (3)

$$D_c = 10, D_f = 10, D_s = 15$$
$$\overline{n}_i = 0$$

$$R_c^{'}(1) = 10 \qquad R_f^{'}(1) = 10 \qquad R_s^{'}(1) = 15$$

$$X_0(1) = 1/35 = 0.028571$$

$$\overline{n}_c(1) = 0.028571 * 10 = 0.28571$$
$$\overline{n}_f(1) = 0.28571 \qquad \overline{n}_s(1) = 0.428571$$

$$R_c^{'}(2) = 10 * 1.28571 = 12.8571 \qquad R_s^{'}(2) = 15 * 1.428571$$
$$R_f^{'}(2) = 12.8571 \qquad = 21.4286$$

$$X_0(2) = 2/(12.86 + 12.86 + 21.43) = 2/47.14276 = 0.0424$$

$$\overline{n}_c(2) = 0.5455 \qquad \overline{n}_f(2) = 0.5455 \qquad \overline{n}_s(2) = 0.9090$$

then: $R_i = R_i^{'} / V_i \qquad U_i = X_i S_i$

Decomposition, Hierarchical Modeling

- Divide and conquer
  - simplicity
  - solvability
- System in Fig. 5.6 [Men 94]
- Exact solution, Tbl 5.1
- Flow equivalent service center for the I/O subsystem
  - load dependent server

how to find these?

Load	S
1	8.25 s
2	6.34 s
3	5.77 s
…	...

Load Equivalent Server

- 1) Short circuit subsystem from rest of the model
  - Fig. 5.7 [Men 94]
- 2) Solve new model for mpl=1, 2, 3, ….

Get	$X_0(1) = 7.27$	->	$S_{I/O}(1) = 8.25$ s
	$X_0(2) = 9.46$	->	$S_{I/O}(2) = 6.34$ s
	$X_0(3) = 10.40$	->	$S_{I/O}(3) = 5.77$ s
	…		...

- 3) Use subsystem as load dependent server in original model (instead of the subsystem)
  - Fig 5.8
- 4) Solve new original model: Fig 5.9, Tbl 5.2

MVA with Load Dependent Servers

- New (3rd) method to compute R_i (or R'_i) device population probabilities, and device utilizations
- Box 36.1 from [Jain 91]

Hierarchical Modeling Summary

- Split model into upper level and lower level models (or even more levels)
- "Shortcut" lower level model, solve $X_k(\mathbf{N})$ for all relevant populations $\mathbf{N}$
  - MVA, convolution, Markov, simulation, …
- Replace sub-model in original model with a load-dependent server: $S_k(\mathbf{N}) = 1/X_k(\mathbf{N})$
- Solve new model for all relevant perf. measures
  - MVA, convolution, Markov, ABA, BJB, simulation, …

BJB - Balanced Job Bounds

- Original model D_c=10, D_f=10, D_s=15 (secs)
- Slower bound model D_c=15, D_f=15, D_s=15 (secs)
- Faster bound model D_c=11.7, D_f=11.7, D_s=11.7
- Solve with MVA $V_iS_i=D_i=D$

ave max

$$\bar{n}_i(N) = N / K \quad (\forall i)$$

$$R'_i(N) = D\left[1 + \bar{n}_i(N-1)\right] = D\left[1 + \frac{N-1}{K}\right] \qquad (\forall i)$$

$$X_0(N) = \frac{N}{\sum_k R'_i(N)} = \frac{N}{K * D\left[1 + \frac{N-1}{K}\right]} = \frac{N}{D\left(K + N - 1\right)}$$

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PMVA

- Purdue Mean Value Analyzer
  - Jeff Brumfield, Purdue Univ, 1981
- PMVA
  - Fig Y1: textual input file
  - Fig Y2: textual output file
  - PMVA syntax slides (2)
  - PMVA example ex1.pmva

Example: Hierarchical Solution with PMVA

- Original Model — Fig. 8.2 (a) [Men 94]
- Solve orig. model — fig.8.2.out [Kerola]
- Sub-model — Fig. 8.2 (b) [Men 94]
- Composite Model — Fig. 8.2 (c) [Men 94]
- Solve sub-model with PMVA for pop=1,2,3 — fig.8.2a.out
- Define load-dep server in composite model and solve it — fig.8.2b.out
  (compare to orig. model solution)