

Lecture 8 MVA, Hierarchical Models Solution Packages

MVA
Hierarchical Models
PMVA

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Mean Value Analysis (MVA)

- A new way to compute the same results as with convolution
- Can compute results without state space enumeration (have class population enumeration)
- Based on Arrival Theorem: $A_i(N) = \bar{n}_i(N-1)$
 - With N jobs in the system, a job arriving to some server i sees the server as in equilibrium with one job (himself!) removed from the system

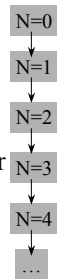
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Arrival Theorem Example

- 2 jobs in closed network: CPU & 2 disks
- Fig. 5.5
- One job (J) in transition (in a “cloud”)
- Look at steady state job distribution for 1 job in the network
 - (e.g., 0.4 at CPU, .3 at both disks)
- Job J sees in average 0.4 jobs at CPU when it arrives there



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Single Class MVA Algorithm

starting point: $A_i(N) = \bar{n}_i(N-1)$ $N=1$ $\bar{n}_i(0) = 0$

residence time

$$R'_i(N) = \begin{cases} D_i[1 + A_i(N)] & \text{arrival} \\ D_i & \text{thm} \end{cases} = D_i[1 + \bar{n}_i(N-1)] \quad \begin{matrix} \text{queueing device} \\ \text{delay device} \end{matrix}$$

$$X_0(N) \stackrel{\text{Little}}{=} \frac{N}{Z + \sum_k R'_k(N)} \quad (\text{response time law})$$

$$\bar{n}_i(N) \stackrel{\text{Little}}{=} \frac{X_i(N) R'_i(N)}{X_0(N)} = \frac{X_i(N) R'_i(N)}{X_0(N)}$$

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MVA Example ⁽³⁾ $D_c = 10, D_f = 10, D_s = 15$ $\bar{n}_i = 0$

$R'_c(1) = 10$	$R'_f(1) = 10$	$R'_s(1) = 15$
$X_0(1) = 1/35 = 0.028571$		
$\bar{n}_c(1) = 0.028571 * 10 = 0.28571$	$\bar{n}_f(1) = 0.28571$	$\bar{n}_s(1) = 0.428571$
$R'_c(2) = 10 * 1.28571 = 12.8571$	$R'_f(2) = 12.8571$	$R'_s(2) = 15 * 1.428571 = 21.4286$
$X_0(2) = 2/(12.86 + 12.86 + 21.43) = 2/47.14276 = 0.0424$		
$\bar{n}_c(2) = 0.5455$	$\bar{n}_f(2) = 0.5455$	$\bar{n}_s(2) = 0.9090$

then: $R_i = R'_i / V_i$ $U_i = X_i S_i$

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Decomposition, Hierarchical Modeling

- Divide and conquer
 - simplicity
 - solvability
- System in Fig. 5.6 [Men 94]
- Exact solution, Tbl 5.1
- Flow equivalent service center for the I/O subsystem
 - load dependent server

how to
find these?

Load	S
1	8.25 s
2	6.34 s
3	5.77 s
...	...

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Load Equivalent Server

- 1) Short circuit subsystem from rest of the model
 - Fig. 5.7 [Men 94]
- 2) Solve new model for $mpl=1, 2, 3, \dots$

Get	$X_0(1) = 7.27$	$\rightarrow$	$S_{IO}(1) = 8.25$ s
	$X_0(2) = 9.46$	$\rightarrow$	$S_{IO}(2) = 6.34$ s
	$X_0(3) = 10.40$	$\rightarrow$	$S_{IO}(3) = 5.77$ s
	...		...

- 3) Use subsystem as load dependent server in original model (instead of the subsystem)
 - Fig 5.8
- 4) Solve new original model: Fig 5.9, Tbl 5.2

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MVA with Load Dependent Servers

- New (3rd) method to compute R_i (or R'_i) device population probabilities, and device utilizations
- Box 36.1 from [Jain 91]

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Hierarchical Modeling Summary

- Split model into upper level and lower level models (or even more levels)
- "Shortcut" lower level model, solve $X_k(N)$ for all relevant populations N
 - MVA, convolution, Markov, simulation, ...
- Replace sub-model in original model with a load-dependent server: $S_k(N) = 1/X_k(N)$
- Solve new model for all relevant perf. measures
 - MVA, convolution, Markov, ABA, BJB, simulation, ...

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BJB - Balanced Job Bounds

- Original model $D_c=10, D_f=10, D_s=15$ (secs)
- Slower bound model $D_c=15, D_f=15, D_s=15$ (secs)
- Faster bound model $D_c=11.7, D_f=11.7, D_s=11.7$
- Solve with MVA $V_i S_i = D_i = D$

$$\bar{n}_i(N) = N / K \quad (\forall i)$$

$$R'_i(N) = D \left[1 + \bar{n}_i(N-1) \right] = D \left[1 + \frac{N-1}{K} \right] \quad (\forall i)$$

$$X_0(N) = \frac{N}{\sum_k R'_k(N)} = \frac{N}{K * D \left[1 + \frac{N-1}{K} \right]} = \frac{N}{D(K+N-1)}$$

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PMVA

- Purdue Mean Value Analyzer
 - Jeff Brumfield, Purdue Univ, 1981
- PMVA
 - Fig Y1: textual input file
 - Fig Y2: textual output file
 - PMVA syntax slides (2)
 - PMVA example ex1.pmva

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Example: Hierarchical Solution with PMVA

- Original Model Fig. 8.2 (a) [Men 94]
- Solve orig. model fig.8.2.out [Kerola]
- Sub-model Fig. 8.2 (b) [Men 94]
- Composite Model Fig. 8.2 (c) [Men 94]
- Solve sub-model with PMVA for pop=1,2,3 fig.8.2a.out
- Define load-dep server in composite model and solve it fig.8.2b.out
(compare to orig. model solution)

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