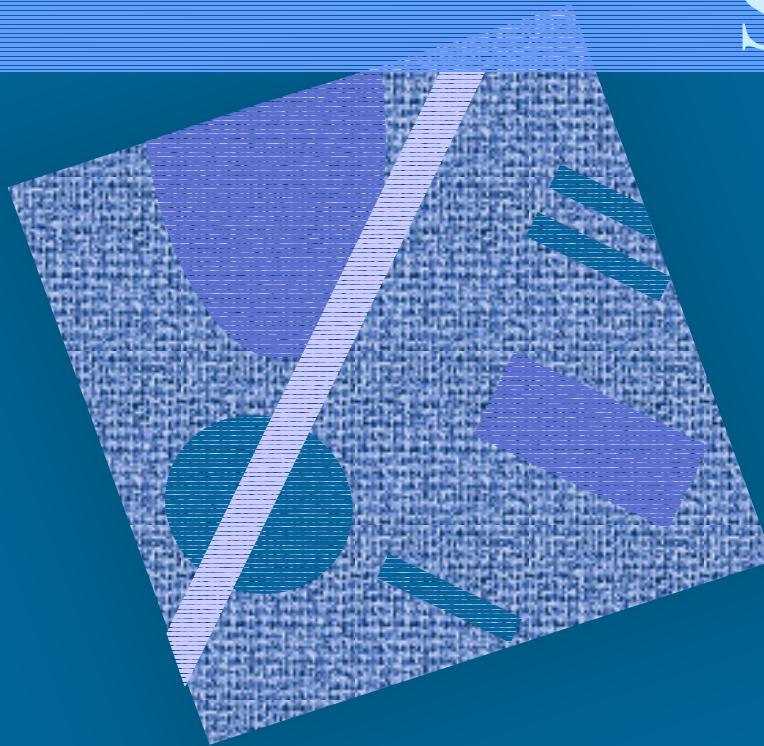


Lecture 8

MVA, Hierarchical Models Solution Packages



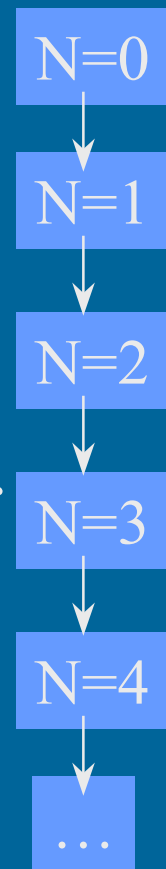
MVA
Hierarchical Models
PMVA

Mean Value Analysis (MVA)

- A new way to compute the same results as with convolution
- Can compute results without state space enumeration (have class population enumeration)
- Based on Arrival Theorem: $A_i(N) = \bar{n}_i(N-1)$
 - With N jobs in the system, a job arriving to some server i sees the server as in equilibrium with one job (himself!) removed from the system

Arrival Theorem Example

- 2 jobs in closed network: CPU & 2 disks
- Fig. 5.5
- One job (J) in transition (in a “cloud”)
- Look at steady state job distribution for 1 job in the network
 - (e.g., 0.4 at CPU, .3 at both disks)
- Job J sees in average 0.4 jobs at CPU when it arrives there



Single Class MVA Algorithm

starting point:

$$A_i(N) = \bar{n}_i(N-1)$$

$$N=1$$

$$\bar{n}_i(0) = 0$$

residence time

$$R'_i(N) = \begin{cases} D_i[1 + A_i(N)] & \text{arrival thm} \\ D_i & \text{queueing device} \\ & \text{delay device} \end{cases}$$

$$X_0(N) \stackrel{\text{Little}}{\underset{\text{system}}{=}} \frac{N}{Z + \sum_k R'_k(N)} \quad (\text{response time law})$$

N+1

$$\begin{aligned} \bar{n}_i(N) &\stackrel{\text{Little}}{\underset{\text{device}}{=}} X_i(N) R_i(N) = X_0(N) \underbrace{V_i}_{X_i(N)} \overbrace{R_i(N)}^{R'_i(N)} \\ &= X_0(N) R'_i(N) \end{aligned}$$

MVA Example

$$D_c = 10, D_f = 10, D_s = 15$$

$$\bar{n}_i = 0$$

$$R'_c(1) = 10$$

$$R'_f(1) = 10$$

$$R'_s(1) = 15$$

$$X_0(1) = \frac{1}{35} = 0.028571$$

$$\bar{n}_c(1) = 0.028571 * 10 = 0.28571$$

$$\bar{n}_f(1) = 0.28571$$

$$\bar{n}_s(1) = 0.428571$$

$$R'_c(2) = 10 * 1.28571 = 12.8571$$

$$R'_f(2) = 12.8571$$

$$R'_s(2) = 15 * 1.428571$$

$$= 21.4286$$

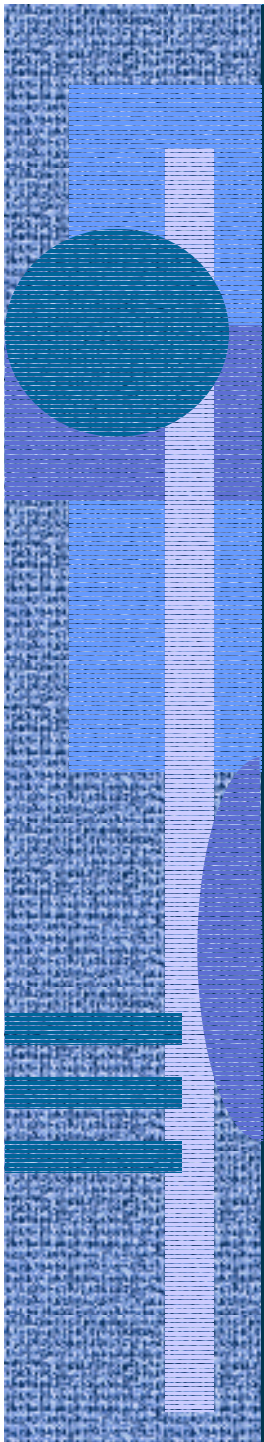
$$X_0(2) = \frac{2}{(12.86 + 12.86 + 21.43)} = \frac{2}{47.14276} = 0.0424$$

$$\bar{n}_c(2) = 0.5455$$

$$\bar{n}_f(2) = 0.5455$$

$$\bar{n}_s(2) = 0.9090$$

then: $R_i = R'_i / V_i \quad U_i = X_i S_i$



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Decomposition, Hierarchical Modeling

- Divide and conquer
 - simplicity
 - solvability
- System in Fig. 5.6 [Men 94]
- Exact solution, Tbl 5.1
- Flow equivalent service center for the I/O subsystem
 - load dependent server

how to
find these?



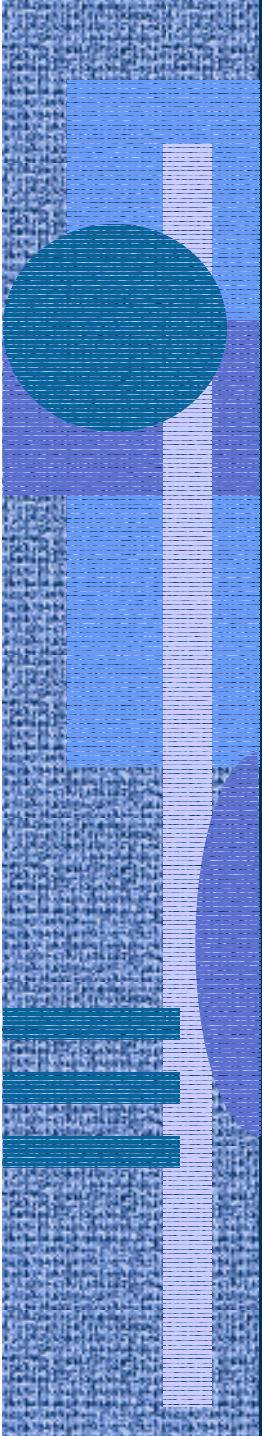
Load	S
1	8.25 s
2	6.34 s
3	5.77 s
...	...

Load Equivalent Server

- 1) Short circuit subsystem from rest of the model
– Fig. 5.7 [Men 94]
- 2) Solve new model for $mpl=1, 2, 3, \dots$

Get	$X_0(1) = 7.27$	->	$S_{I/O}(1) = 8.25 \text{ s}$
	$X_0(2) = 9.46$	->	$S_{I/O}(2) = 6.34 \text{ s}$
	$X_0(3) = 10.40$	->	$S_{I/O}(3) = 5.77 \text{ s}$
	...		...

- 3) Use subsystem as load dependent server in original model (instead of the subsystem)
– Fig 5.8
- 4) Solve new original model: Fig 5.9, Tbl 5.2



MVA with Load Dependent Servers

- New (3rd) method to compute R_i (or R'_i) device population probabilities, and device utilizations
- Box 36.1 from [Jain 91]

Hierarchical Modeling Summary

- Split model into upper level and lower level models (or even more levels)
- "Shortcut" lower level model, solve $X_k(\mathbf{N})$ for all relevant populations $\mathbf{N}$
 - MVA, convolution, Markov, simulation, ...
- Replace sub-model in original model with a load-dependent server: $S_k(\mathbf{N}) = 1/X_k(\mathbf{N})$
- Solve new model for all relevant perf. measures
 - MVA, convolution, Markov, ABA, BJB, simulation, ...

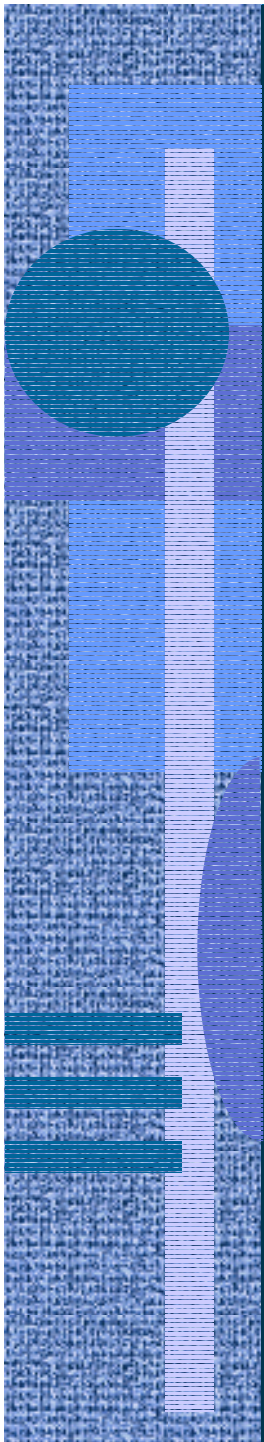
BJB - Balanced Job Bounds

- Original model $D_c=10, D_f=10, D_s=15$ (secs)
- Slower bound model $D_c=15, D_f=15, D_s=15$ (secs)
- Faster bound model $D_c=11.7, D_f=11.7, D_s=11.7$ ↑
ave
- Solve with MVA $V_i S_i = D_i = D$ ↑
max

$$\bar{n}_i(N) = N / K \quad (\forall i)$$

$$R'_i(N) = D \left[1 + \bar{n}_i(N-1) \right] = D \left[1 + \frac{N-1}{K} \right] \quad (\forall i)$$

$$X_0(N) = \frac{N}{\sum_k R'_i(N)} = \frac{N}{K * D \left[1 + \frac{N-1}{K} \right]} = \frac{N}{D(K+N-1)}$$



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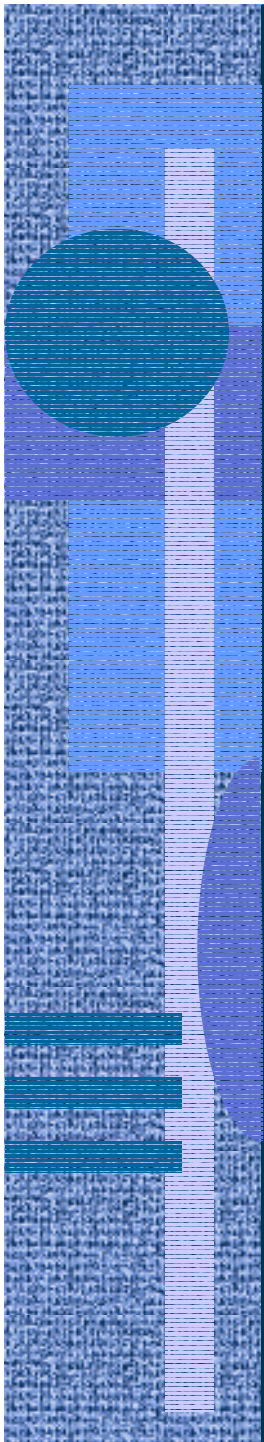
12

PMVA

- Purdue Mean Value Analyzer
 - Jeff Brumfield, Purdue Univ, 1981
- PMVA
 - Fig Y1: textual input file
 - Fig Y2: textual output file
 - PMVA syntax slides (2)
 - PMVA example ex1.pmva

Example: Hierarchical Solution with PMVA

- Original Model Fig. 8.2 (a) [Men 94]
- Solve orig. model fig.8.2.out [Kerola]
- Sub-model Fig. 8.2 (b) [Men 94]
- Composite Model Fig. 8.2 (c) [Men 94]
- Solve sub-model with PMVA for pop=1,2,3 fig.8.2a.out
- Define load-dep server in composite model and solve it fig.8.2b.out
(compare to orig. model solution)



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15