Relational algebra

- Union
- Difference
- Cross product
- Intersection
- Projection
- Selection
- Joins
  - Normal join (theta join, equijoin)
  - Natural join
  - Outer join

Cardinality of results: \( R \circ S \)

- \( c(R) = \) cardinality (number of rows) of \( R \)
- Union: rows of both relations, common ones only once
  - \( c(R \cup S) = c(R) + c(S) - c(R \cap S) \geq \max(c(R), c(S)) \)
- Difference: Rows of \( R \) that are not in \( S \)
  - \( c(R - S) = c(R) - c(R \cap S) \leq c(R) \)
- Intersection: Rows common to \( R \) and \( S \)
  - \( 0 \leq c(R \cap S) \leq \min(c(R), c(S)) \)

Cardinality of results: \( R \circ S \)

- Product: each row of \( R \) connected to each row of \( S \)
  - \( c(R \times S) = c(R) \times c(S) \)
- Projection: pick up distinct values appearing in a column collection
  - \( c(\pi_a (R)) \leq c(R) \)
  - \( c(\pi_a (R)) \leq c(\pi_{a,b} (R)) \)
  - if \( a \) contains a key: \( c(\pi_a (R)) = c(R) \)
  - \( c(\pi_a (R)) = 0 \) only if \( c(R) = 0 \)

Cardinality of results: \( R \circ S \)

- Selection: pick up rows that conform to the selection criteria
  - \( 0 \leq c(\sigma_{a = \text{constant}} (R)) \leq 1 \), when \( k \) is a key of \( R \)
  - \( c(\sigma_{a = \text{constant}} (R)) \leq c(R) \), when \( k \) is a key of \( R \)
  - \( c(\sigma_{a \text{ or } f} (R)) \leq \min(c(\sigma_{a} (R), c(\sigma_{f} (R))) \)
  - \( c(\sigma_{a \text{ or } f} (R)) \leq \max(c(\sigma_{a} (R), c(\sigma_{f} (R))) \)

Cardinality of results: \( R \circ S \)

- Join: row of \( R \) and \( S \) are connected based on connection criteria
  - \( 0 \leq c(R \bowtie_k S) \leq c(R \times S) \)
  - Let \( k \) be the key of \( R \) and \( v \) a foreign key in \( S \) that refers to \( R \)
    - \( c(R \bowtie_{k=v} S) \leq c(S) \), if null values are allowed for \( v \)
    - \( c(R \bowtie_{k=v} S) = c(S) \), if null values are not allowed for \( v \)

Natural join:

- \( c(R \bowtie S) = c(R \times S) \), if \( R \) and \( S \) do not have columns with common name
- mostly natural joins carry out a join based on the equality of foreign key and the corresponding primary key
- If both \( R \) and \( S \) have a common schema, natural join equals to intersection

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Cardinality of results: $R \op S$

- outer join: Rows that drop out in normal join will also be included with 'empty' pair
- $c(R \bowtie_{\text{antil}} S) \geq c(R)$