PWM (positional weight matrix):

$$\begin{matrix} A \\ C \\ G \\ T \end{matrix}\begin{bmatrix} 2/6 & 0 & 1 & 0 & 4/6 & 4/6 \\ 0 & 0 & 0 & 0 & 2/6 & 0 \\ 0 & 5/6 & 0 & 0 & 0 & 2/6 \\ 4/6 & 1/6 & 0 & 1 & 0 & 0 \end{bmatrix} \approx \begin{bmatrix} 0.33 & 0 & 1 & 0 & 0.67 & 0.67 \\ 0 & 0 & 0 & 0 & 0.33 & 0 \\ 0 & 0.83 & 0 & 0 & 0 & 0.33 \\ 0.67 & 0.17 & 0 & 1 & 0 & 0 \end{bmatrix}$$

consensus sequence: *TGATAA*

PSSM (position-specific scoring matrix):

counts:

$$\begin{matrix} A \\ C \\ G \\ T \end{matrix}\begin{bmatrix} 2 & 0 & 6 & 0 & 4 & 4 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 5 & 0 & 0 & 0 & 2 \\ 4 & 1 & 0 & 6 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ pseudocounts:

$$\begin{matrix} A \\ C \\ G \\ T \end{matrix}\begin{bmatrix} 3 & 1 & 7 & 1 & 5 & 5 \\ 1 & 1 & 1 & 1 & 3 & 1 \\ 1 & 6 & 1 & 1 & 1 & 3 \\ 5 & 2 & 1 & 7 & 1 & 1 \end{bmatrix}$$

$\Rightarrow$ PSSM:

$$\begin{matrix} A \\ C \\ G \\ T \end{matrix}\begin{bmatrix} 0.26 & -1.32 & 1.49 & -1.32 & 1 & 1 \\ -1.32 & -1.32 & -1.32 & -1.32 & 0.26 & -1.32 \\ -1.32 & 1.26 & -1.32 & -1.32 & -1.32 & 0.26 \\ 1 & -0.32 & -1.32 & 1.49 & -1.32 & -1.32 \end{bmatrix}$$

assuming $q_a = 0.25$ for all a

$log(p_{ai}/q_a) = log((\frac{1}{10})/(\frac{1}{4})$
$= log(\frac{1}{10} \cdot 4) = log\frac{2}{5} \approx -1.32$

$log(\frac{2}{10} \cdot 4) = log\frac{4}{5} \approx -0.32$
$log(\frac{3}{10} \cdot 4) = log\frac{6}{5} \approx 0.26$
$log(\frac{5}{10} \cdot 4) = log2 = 1$
$log(\frac{6}{10} \cdot 4) = log\frac{12}{5} \approx 1.26$
$log(\frac{7}{10} \cdot 4) = log\frac{14}{5} \approx 1.49$

sequence logo heights:

height of a at column i is: $p_{ai} \cdot E(S_i) = p_{ai} \cdot \sum_a p_{ai} \cdot log(p_{ai}/q_a)$

$$
\begin{aligned}
E(S_1) &= \tfrac{3}{10} \cdot log\tfrac{6}{5} + \tfrac{1}{10} \cdot log\tfrac{2}{5} + \tfrac{1}{10} \cdot log\tfrac{2}{5} + \tfrac{5}{10} \cdot log 2 \\
&\approx \tfrac{3}{10} \cdot 0.26 + \tfrac{1}{10} \cdot (-1.32) + \tfrac{1}{10} \cdot (-1.32) + \tfrac{5}{10} \cdot 1 \\
&\approx 0.314
\end{aligned}
$$

$$
\begin{aligned}
E(S_2) &\approx \tfrac{1}{10} \cdot (-1.32) + \tfrac{1}{10} \cdot (-1.32) + \tfrac{6}{10} \cdot 1.26 + \tfrac{2}{10} \cdot (-0.32) \approx 0.428 \\
E(S_3) &\approx \tfrac{7}{10} \cdot 1.49 + \tfrac{1}{10} \cdot (-1.32) + \tfrac{1}{10} \cdot (-1.32) + \tfrac{1}{10} \cdot (-1.32) \approx 0.647 \\
E(S_4) &= E(S_3) \approx 0.647 \\
E(S_5) &= E(S_1) \approx 0.314 \\
E(S_6) &= E(S_1) \approx 0.314
\end{aligned}
$$

$$
\begin{aligned}
p_{A1} \cdot E(S_1) &\approx \tfrac{3}{10} \cdot 0.314 \approx 0.09 & p_{A2} \cdot E(S_2) &\approx \tfrac{1}{10} \cdot 0.428 \approx 0.04 \\
p_{C1} \cdot E(S_1) &\approx \tfrac{1}{10} \cdot 0.314 \approx 0.03 & p_{C2} \cdot E(S_2) &\approx \tfrac{1}{10} \cdot 0.428 \approx 0.04 \\
p_{G1} \cdot E(S_1) &\approx \tfrac{1}{10} \cdot 0.314 \approx 0.03 & p_{G2} \cdot E(S_2) &\approx \tfrac{6}{10} \cdot 0.428 \approx 0.26 \\
p_{T1} \cdot E(S_1) &\approx \tfrac{5}{10} \cdot 0.314 \approx 0.16 & p_{T2} \cdot E(S_2) &\approx \tfrac{2}{10} \cdot 0.428 \approx 0.09 \\
&\dots
\end{aligned}
$$

heights:

$$
\begin{array}{c}
A \\ C \\ G \\ T
\end{array}
\begin{bmatrix}
0.09 & 0.04 & 0.45 & 0.07 & 0.16 & 0.16 \\
0.03 & 0.04 & 0.07 & 0.07 & 0.09 & 0.03 \\
0.03 & 0.26 & 0.07 & 0.07 & 0.03 & 0.09 \\
0.16 & 0.09 & 0.07 & 0.45 & 0.03 & 0.03
\end{bmatrix}
$$