

Mathematical Modelling for Computer Networks

Spring 2013

Exercise 4: Due on 12th April 2013.

Write your answers to the questions briefly and clearly. Please bring a printout (or a handwritten copy) of your answers to the class. You may refer to the book Introduction to Probability by Grinstead and Snell (http://www.dartmouth.edu/~chance/teaching_aids/books_articles/probability_book/book.html)

1. A family has two children. What is the conditional probability that both are boys given that at least one of them is a boy?
2. Each of three guests in a party throws his/her hat into the centre of a room. The hats are mixed up and each randomly picks up a hat. What is the probability that none of the three guests winds up with his/her own hat?
3. In a multiple choice test a student either knows or guesses the answer in answering a question. Let p be the probability that the student knows the answer and $1-p$ is the probability that he/she guesses. Assuming that a student who guesses at the answer will get the correct answer with probability $1/m$ where m is the number of multiple choice alternatives. What is the conditional probability that a student knew the answer to a question given that he/she answered it correctly?
4. Let X be a binomial random variable with parameters n and p . Calculate the mean and variance of X using the mean and variance formula for a random variable.
5. Let X be a Poisson random variable with parameter λ . Calculate the mean and variance of X using the mean and variance formula for a random variable.
6. Urn models are widely used in probabilistic modelling. This question is an example. In an urn there are R red balls and B black balls. What is the probability of drawing j red balls in a total of n draws in each of the following two cases: (a) after each draw we replace the drawn ball, mix the balls up and draw at random again and (b) after each draw the drawn ball is not replaced.
7. The number of typographical errors in a page of a book is Poisson distributed with parameter $\lambda = 1$. Calculate the probability of the events (a) there is at least one error in a page and (b) there are no errors in a page.