Mathematical Modelling for Computer Networks- Part II Autumn 2013

Exercise 4: Due on 22nd November 2013.

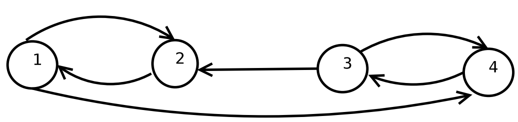
Attempt all the questions. Write your answers to the questions briefly and clearly. Please bring a printout (or a handwritten copy) of your answers to the class. We discuss the related topics in detail in the exercise class.

1. State the definition of a Markov process and give an intuitive interpretation of the definition.

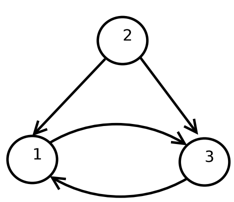
2. In a graph $G(V, E)$, define the graph $\bar{G}(V, \bar{E})$ which has the same vertex set V as G but has an edge $(i, j) \in \bar{E}$ iff $(i, j) \notin E$. What is the relationship between the independent sets and cliques of the graphs G and $\bar{G}$?

3. Identify the recurrent classes of states of the following Markov chains

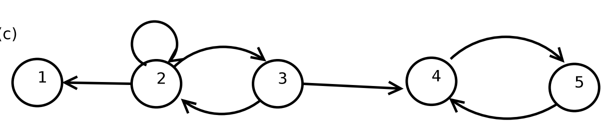
(a)



(b)



(c)



4. Gambler's ruin: A gambler G wins €1 in each round with probability p or losses €1 with probability 1-p. Different rounds are assumed to be independent. The gambler keeps on playing until he wins an amount €m or loses all his money. What is the probability that he eventually wins the target amount or loses all his fortune?

5. Packets that arrive in a communication network are stored in a buffer before they are transmitted. Assume that the buffer has storage capacity m and the time is slotted. In each time slot, exactly one of the following events happens.
   (a)A new packet arrives with a probability $b > 0$
   (b) An existing packet completes transmission with probability $d > 0$
   (c) No new packet arrives and no existing packet completes transmission, this happens with a probability $1 - b - d$ if there is at least one packet in the node and with probability $1 - b$ otherwise. Draw the Markov Chain representation of this question.