Mathematical Modelling for Computer Networks- Part II Autumn 2013

Exercise 5: Due on 29th November 2013.

Attempt all the questions. Write your answers to the questions briefly and clearly. Please bring a printout (or a handwritten copy) of your answers to the class. We discuss the related topics in detail in the exercise class.

1. Fill in the calculations to obtain the result of Theorem 2.3(b) (page 4 of the book 'Scheduling and Congestion Control for Wireless and Processing Networks' by L. Jiang and J. Walrand, Morgan & Claypool publishers, 2010).

2. Theorem 2.5 (page 7 of the above book) shows that MWM scheduling makes queues positive recurrent. Understand the proof of this theorem.

3. The invariant distribution of (an irreducible) Markov chain is a probability vector π such that $\pi = \pi\mathbf{P}$ where $\mathbf{P}$ is one-step transition matrix of the Markov chain. Calculate π for the 2-state Markov chain

$$\mathbf{P} = \begin{bmatrix} \alpha & 1-\alpha \\ \beta & 1-\beta \end{bmatrix}$$

   Evaluate π_0 for the case $\alpha = 0.7$ and $\beta = 0.7$

4. Two states i and j that are accessible from each other ($i \to j$ and $j \to i$) *communicate* with each other. Notation ($i \leftrightarrow j$). Show that the communication relation is an equivalence relation i.e, it is reflexive, symmetric and transitive.

5. Determine if the Markov chain with the given matrix P is irreducible.

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

6. Let a Markov chain with states 0, 1, 2, 3 have the transition matrix P as follows.

$$P = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Determine which states are transient and which are recurrent