Mathematical Modelling of Computer Networks: Part II

Module 2: Wireless Scheduling

Lecture 2: Wireless Scheduling

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Wireless Scheduling - Today's lecture

- Recap of Lyapunov theorem for Markov chain stability
- Maximum Weighted Matching (MWM) scheduling
- Definition of MWM Scheduling
- Proof of the stability of MWM scheduling
- Salient features of CSMA Scheduling
- Summary

Slides are based on the second chapter of the book 'Scheduling and Congestion Control for Wireless and Processing Networks' by L. Jiang and J. Walrand, Morgan & Claypool publishers, 2010

Maximum Weighted Matching (MWM) scheduling

- In lecture 1, we saw that when the arrival rates are strictly feasible, there is schedule that makes the queues positive recurrent
- A randomized scheduling that was proposed for this problem required knowing the arrival rates
- Next we describe an algorithm that does not require the knowledge of arrival rates
- The scheduling algorithm we describe now is MWM scheduling

Definition of MWM scheduling

- MWM scheduling identifies the independent set of nodes with the largest value for the sum of the backlogs of the nodes in that set and serves them.
- (If there are two or more independent sets of nodes with the same largest value for the sum of their nodes, it serves any of them)
- So for our example network, the MWM algorithm serves queue 2 if the backlog of that queue is larger than the sum of the backlogs of queues 1 and 3; otherwise it serves queues 1 and 3.
- This means the algorithm serves the independent set with the largest sum of backlogs4

MWM and Positive recurrence - (1/5)

- Theorem: Assuming the arrival rates to be strictly feasible and with finite variance, MWM algorithm makes the queues stable i.e, it has positive recurrence property.
- To establish this,we use the approach like the one we used previously.
- Let $X_i(n)$ be the queue length of node i at time n (so i = 1, 2,3 and n = 0, 1, 2, ...) . let $X(n) = (X_1(n), X_2(n), X_3(n))$ be the vector of queue lengths.
- $$\text{Let } V(X(n)) = \frac{1}{2}[X_1^2(n) + X_2^2(n) + X_3^2(n)]$$
- We want to show that under MWM scheduling, $V(X(n))$ is a Lyapunov function of the Markov chain $X(n)$

MWM and Positive recurrence (2/5)

- Proceeding in the same way as before (refer to Lecture 1), we get the expression
  $X_i^2(n+1) - X_i^2(n)$

  $= A_1^2(n) + Z_i^2(n) + 2X_i(n)A_i(n) - 2X_i(n)Z_i(n) - 2A_i(n)Z_i(n)$

  $= A_1^2(n) + Z_i^2(n) + 2X_i(n)A_i(n) - 2X_i(n)S_i(n) - 2A_i(n)Z_i(n)$

- Noting that $S_i(n)$ is a function of $X(n)$ determined by MWM algorithm, we get

$$E(V(X(n+1) - V(X(n)|X(n)) \leq \beta + \sum_{i=1}^{3}(\lambda_i - S_i(n))X_i(n)$$

- Note that this inequality is quite the same as the one we obtained earlier with $S_i(n)$ in the place of p_i

MWM and Positive recurrence (3/5)

- To prove the Lyapunov stability of MWM, it is enough to show that the expression $\beta + \sum_{i=1}^{3}(\lambda_i - S_i(n))X_i(n)$ the right-hand side (RHS) of the inequality below is less than $-\epsilon$ outside a finite set

$$E[V(X(n+1) - V(X(n)|X(n)] \leq \beta + \sum_{i=1}^{3}(\lambda_i - S_i(n))X_i(n)$$

- So MWM algorithm chooses a value of $S_i(n) = i = 1, 2, 3$ that maximizes $\sum_{i=3}^{3}(\lambda_i - S_i(n))X_i(n)$ subject to the scheduling constraints
- The maximum value must then be larger than $pX_1(n) + (1-p)X_2(n) + pX_3(n)$ where p is the probability such that the mean arrival rates, λ_i, $i = 1, 2, 3$ are strictly feasible

MWM and Positive recurrence (4/5)

- The feasibility constraints say that the mean arrival rates should satisfy the constraints $\lambda_1 + \lambda_2 < 1$ and $\lambda_2 + \lambda_3 < 1$
- These constraints imply that there is some $p \in [0, 1]$ such that

$$\lambda_1 + \lambda_2 < 1 \text{ and } \lambda_2 + \lambda_3 < 1$$

- The maximum is either $X_1(n) + X_3(n)$ or $X_2(n)$ and this maximum is larger than any convex combination of these two values. Hence,

$$E[V(X(n+1) - V(X(n)|X(n)]$$

$$\leq \beta + (\lambda_1 - p)X_1(n) + (\lambda_2 - (1-p))X_2(n) + (\lambda_3 - p)X_3(n)$$

- In the proof of the theorem given in lecture 1, we showed that the right-hand side is less ϵ when $X(n)$ is outside a finite set

MWM and Positive recurrence (5/5)

- In conclusion we have shown that when the arrival rates are strictly feasible and have finite variance, the Markov chain corresponding to MWM scheduling makes the queues positive recurrent and so MWM scheduling stabilizes the queues
- Furthermore, it is a basic result that MWM scheduling is throughput optimal which means that if any scheduling algorithm can stabilize the queues (subject to the scheduling constraints). MWM scheduling can also stabilize the queues
- Though this is a theoretically interesting result, it is difficult to implement in practice especially in large wireless networks
- Distributed randomized algorithms such as CSMA (Carrier Sense Multiple Access) algorithm approximate the performance of MWM algorithm while being attractive for practical implementation