A Las Vegas algorithm is a randomized algorithm that always returns the right answer, but whose running time is random.

A Monte Carlo algorithm is a randomized algorithm that may fail or return an incorrect answer. The running time often does not depend on the random choices made.

Any Monte Carlo algorithm whose answer can be verified to be correct or incorrect can be turned into a Las Vegas algorithm by running it repeatedly until it succeeds.

Denote by \( \tilde{F} \) the corrupted version of \( F \).

Algorithm:
1. Draw \( X \) from \( 1, 2, \ldots, n-1 \) uniformly at random.
2. Let \( Y = Z - X \mod n \).
3. Let \( R = (\tilde{F}(X) + \tilde{F}(Y)) \mod n \).
4. Return \( R \).

Analysis:
By the given properties of \( F \),

\[
\Pr(R = F(Z)) = \Pr(R = (F(X) + F(Y)) \mod n) = 1 - \Pr(\tilde{F}(X) \neq F(X) \text{ or } \tilde{F}(Y) \neq F(Y))
\]

(Union bound)

\[
\geq 1 - \left[ \Pr(\tilde{F}(X) \neq F(X)) + \Pr(\tilde{F}(Y) \neq F(Y)) \right] = \frac{3}{5} > \frac{1}{2}.
\]
Let \((Y_1, Y_2, \ldots, Y_n)\) be the permutation of input numbers.

Let \(X_k\) be the number of swaps when handling \(Y_k\).

For \(j \in \{0, 1, \ldots, k\}\) we have \(\Pr(X_k = j) = \frac{1}{k}\),

since \((Y_1, \ldots, Y_n)\) is a random permutation of \(\{Y_1, \ldots, Y_k\}\).

Thus, the expected number of swaps is

\[
E\left[\sum_{k=1}^{n} X_k\right] = \sum_{k=1}^{n} E[X_k] = \sum_{k=1}^{n} \frac{k-1}{k} = \frac{n(n-1)}{2} = \frac{n(n-1)}{4}.
\]

Let \(X = \sum_{i=1}^{n} X_i\), with \(X_i = 1\) if \(i\) is not isolated,

and \(X_i = 0\) if \(i\) is isolated.

Clearly it suffices to prove \(\Pr(X \geq 1) \to 1\) as \(n \to \infty\).

The 2nd moment method:

\[
\Pr(X = 0) \leq \text{Var}[X] / E[X]^2 \leq \frac{E[X^2]}{E[X]^2} = \frac{1}{\theta},
\]

we have

\[
E[X] = \sum_{i=1}^{n} E[X_i] = \sum_{i=1}^{n} \Pr(X_i = 1) = n \cdot \theta^{n-1},
\]

\[
E[X^2] = \sum_{i=1}^{n} E[X_i X_j] + \sum_{i=1}^{n} E[X_i^2] = \sum_{i=1}^{n} \Pr(X_i = 1 \text{ and } X_j = 1) + \sum_{i=1}^{n} \Pr(X_i = 1) = (n^2 - n) \theta^{2(n-1)} + n \cdot \theta^{2(n-1)} - 1 \leq \frac{1}{4} + \frac{1}{n^2} < 1 \to 0.
\]

Thus \(\Pr(X = 0) = \frac{(n-1) \theta^{2(n-2)} + 1}{n \theta^{2(n-1)} - 1} \to 0\)

since \(q = 1-p \to 0\) and \(n \cdot q^n = n/1 - \frac{q^n}{n} \to 0\).

\[
\lim_{n \to \infty} n \cdot q^n = \lim_{n \to \infty} \frac{n}{1 - \frac{q^n}{n}} = \lim_{n \to \infty} n \cdot \exp(-qn) = n^{-1} \to 0.
\]

\[
\Pr(X \geq 1) \to 1 \text{ as } n \to \infty.
\]