You are allowed to bring with you one 2-sided paper sheet of notes. Calculators, phones, or other smart devices are not allowed.

Please solve all the four problems given below. Show your work—partial credits will be given. Be clear and neat. Use either English, Swedish, or Finnish. You have two and a half hours in total—plan your time usage! Include your name and identity number, and the name of the course in every paper sheet you return—you need not use a separate sheet for each problem. Number the problems clearly.

Note: To give an algorithm or scheme means not only describing it but also proving that it has the claimed properties (correctness, running time, approximation guarantee, etc.).

1. (max 12 points)
   What are Las Vegas and Monte Carlo algorithms? Define and compare using at most 100 words in total.

2. (max 12 points)
   We have a function \( F : \{0, \ldots, n-1\} \to \{0, \ldots, m-1\} \), where \( n \) and \( m \) are positive integers. We know that, for \( 0 \leq x, y \leq n-1 \), \( F((x+y) \mod n) = (F(x)+F(y)) \mod m \).
   The only way we have for evaluating \( F \) is to use a lookup table that stores the values of \( F \). Unfortunately, an Evil Adversary has changed the value of one fifth of the table entries when we were not looking.
   Describe a simple randomized algorithm that, given an input \( z \), outputs a value that equals \( F(z) \) with probability at least \( 1/2 \). Your algorithm should work for every value of \( z \), regardless of what values the Adversary changed. Your algorithm should use as few lookups and as little computation as possible.

3. (max 12 points)
   *Linear insertion* sort can sort an array of numbers in place. The first and second numbers are compared; if they are out of order, they are swapped so that they are in sorted order. The third number is then placed in the appropriate place in the sorted order. It is first compared with the second; if it is not in the proper order, it is swapped and compared with the first. Iteratively, the \( k \)th number is handled by swapping it downward until the first \( k \) numbers are in sorted order. Determine the expected number of swaps that need to be made with a linear insertion sort when the input is a random permutation of \( n \) distinct numbers.

4. (max 12 points)
   Consider a graph in \( G_{n,p} \), with \( p = c \ln n/n \). Use the second moment method or the conditional expectation inequality to prove that if \( c < 1 \) then, for any constant \( \varepsilon > 0 \) and for sufficiently large \( n \), the graph has isolated vertices with probability at least \( 1-\varepsilon \).