1. **False**  b) **True**  c) **False**  d) **True**  e) **False**  f) **True**

Let $G = (V, E)$ be a graph as in the claim.

By Theorem 7.13 (book) the stationary distribution $\pi$ of the chain $\pi$ is given by

$$\pi_v = \frac{d(v)}{2|E|}, \quad \forall v \in V,$$

where $d(v)$ is the degree of $v$ in $G$.

If $(u, v) \in E$, then $P_{uv} = \frac{1}{d(u)}$ and $P_{vu} = \frac{1}{d(v)}$ and

$$\pi_v P_{uv} = \frac{1}{2|E|}, \quad d(v) = \frac{1}{2|E|} = \frac{1}{d(u)} = \pi_u P_{uv}.$$

Otherwise $P_{uv} = P_{vu} = 0$, and thus $\pi_v P_{uv} = 0 = \pi_u P_{vu}$.

Then $\pi_v P_{uv} = \pi_u P_{vu}$ if $u, v \in V$, meaning the chain is time reversible.

2. Let $W_1, W_2, \ldots, W_n$ be the edge weights ordered in non-decreasing order. The total weight of the minimum spanning tree is clearly at least

$$W_1 + W_2 + \ldots + W_{n-1}.$$

Thus its expected value is at least

$$E[W_1 + W_2 + \ldots + W_{n-1}] = \sum_{k=1}^{n-1} E[W_k].$$

(by Lemma 8.3, book)

$$= \sum_{k=1}^{n-1} \frac{k}{1 + \binom{k}{2}}$$

$$= \frac{\binom{n}{2}}{1 + \binom{n}{2}}$$

$$+ \left(1 - \frac{1}{1 + \binom{n}{2}}\right).$$
Consider a graph $G = (V, E)$. Label the edges as $e_1, e_2, \ldots, e_m$. Let $G_j = (V, e_{1j}, \ldots, e_{jj})$ and let $M_j$ be the set of matchings of $G_j$. An $M_j = 1$, the number of matchings of $G$ is given by

$$ |M_j| = R^{-1} \quad \text{where} \quad R = \prod_{j=2}^{m} R_j \quad \text{with} \quad R_j = \frac{|M_{j-1}|}{|M_j|}.$$ 

Because $M_{j-1} \leq M_j$ and $M_j \leq M_{j-1}$, we have $|M_{j-1}| \leq |M_j|$. Moreover, if $e_j$ has no edge intersecting $\bigcap_{i=1}^{j-1} e_i$, then $\frac{1}{2} \leq R_j \leq 1$ for $j = 2, \ldots, m$.

By Theorem 10.1 (book), we obtain for each $R_j$ an $(3/2m, 1/m)$-approximation $R_j$ by using

$$ \tilde{R}_j = \frac{\ln(m/\delta)}{(3/2m)^j} = \Theta(m/\ln m) \cdot (\frac{1}{\epsilon})^2$$

samples from the uniform distribution over $M_j$.

In total, this is $\Theta(m^3 \ln m)$ samples and yields a PTAS.

More, by the union bound, with prob. at least $1 - \delta$, we have

$$ |\tilde{R}_j - R_j| \leq \frac{\epsilon}{2m}, \quad \Rightarrow \quad 1 - \frac{\epsilon}{2m} \leq \frac{\tilde{R}_j}{R_j} \leq 1 + \frac{\epsilon}{2m},$$

implying that for $\tilde{R} = \prod_{j=2}^{m} \tilde{R}_j$, that

$$ (1 - \frac{\epsilon}{2m})^m \leq (1 - \frac{\epsilon}{2m})^{m-1} \leq \tilde{R}/R \leq (1 + \frac{\epsilon}{2m})^{m-1} \leq (1 + \frac{\epsilon}{2m})^m,$$

which further implies

$$ 1 - 3 \leq \frac{(1 - \frac{\epsilon}{2m})^m}{(2m - \frac{\epsilon}{2m} + 3)} \leq \frac{R/\tilde{R}}{1 - 3} \leq 1 + 3.$$