1. **Exercise 7.2:** Consider the two-state Markov chain with the following transition matrix.

\[
P = \begin{bmatrix}
p & 1-p \\
1-p & p
\end{bmatrix}.
\]

Find a simple expression for \( P_{0,0}^t \).

2. **Exercise 7.5:** Prove that if one state in a communicating class is transient (respectively, recurrent) then all states in that class are transient (respectively, recurrent).

3. **Exercise 7.12:** Let \( X_n \) be the sum of \( n \) independent rolls of a fair die. Show that, for any \( k \geq 2 \),

\[
\lim_{n \to \infty} \Pr(X_n \text{ is divisible by } k) = \frac{1}{k}.
\]

4. **Exercise 7.13:** Consider a finite Markov chain on \( n \) states with stationary distribution \( \bar{\pi} \) and transition probabilities \( P_{i,j} \). Imagine starting the chain at time 0 and running it for \( m \) steps, obtaining the sequence of states \( X_0, X_1, \ldots, X_m \). Consider the states in reverse order, \( X_m, X_{m-1}, \ldots, X_0 \).

a) Argue that given \( X_{k+1} \), the state \( X_k \) is independent of \( X_{k+2}, X_{k+3}, \ldots, X_m \). Thus the reverse sequence is Markovian.

b) Argue that for the reverse sequence, the transition probabilities \( Q_{i,j} \) are given by

\[
Q_{i,j} = \frac{\bar{\pi}_i P_{j,i}}{\pi_i}.
\]

c) Prove that if the original Markov chain is time reversible, so that \( \pi_i P_{i,j} = \pi_j P_{j,i} \), then \( Q_{i,j} = P_{i,j} \). That is, the states follow the same transition probabilities whether viewed in forward order or reverse order.
5. **Exercise 7.19**: Consider the gambler’s ruin problem, where a player plays until they lose $l_1$ dollars or win $l_2$ dollars. Prove that the expected number of games played is $l_1/l_2$.

6. **Exercise 7.18**:
   a) Consider a random walk on the 2-dimensional integer lattice, where each point has four neighbors (up, down, left, and right). Is each state transient, null recurrent, or positive recurrent? Give an argument.
   b) Answer the problem in (a) for the 3-dimensional integer lattice.

7. **Exercise 7.26**: Let $n$ equidistant points be marked on a circle. Without loss of generality, we think of the points as being labeled clockwise from 0 to $n-1$. Initially, a wolf begins at 0 and there is one sheep at each of the remaining $n-1$ points. The wolf takes a random walk on the circle. For each step, it moves with probability 1/2 to one neighboring point and with probability 1/2 to the other neighboring point. At the first visit to a point, the wolf eats a sheep if there is still one there. Which sheep is most likely to be the last eaten.