UNIVERSITY OF HELSINKI
Department of Computer Science
P.O. Box 68 (Gustaf Hällströmin katu 2b)
00014 UNIVERSITY OF HELSINKI

581330-2 Models for Programming and Computing

Exercise 1 (24.–28.1.2005)

1. Write out the following sets:
   (a) $(\{1,2,2,3\} \setminus \{2,5,5\}) \cup \{1,7\}$.
   (b) $\mathcal{P}(\{a,b,c\}) \setminus \mathcal{P}(\{b,c,d\})$.
   (c) $\mathcal{P}(\mathcal{P}(\emptyset)) \times \{0,1\}$.
2. Prove that
   (a) If A and B are finite sets and $A \cap B = \emptyset$, then $A \cup B$ is finite. *Hint: Use the definition of finiteness: a set A is finite if there is a bijection $f\colon \{1,\ldots,n\} \to A$ for some $n \in \mathbb{N}$.*
   (b) If A and B are finite, then $A \cup B$ is finite. *Hint: Use part (a).*
   (c) If A is infinite and B is finite, then $A \setminus B$ is infinite. *Hint: Use proof by contradiction.*
3. Use induction to show that
   (a) $\sum_{i=0}^{n} 1 = n + 1$.
   (b) $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$.
   (c) $\sum_{i=0}^{n} 3^i = \frac{3^{n+1}-1}{2}$.

   Use the above to determine $|\{0\}^{\leq k}|$, $|\{0,1\}^{\leq k}|$ and $|\{0,1,2\}^{\leq k}|$.
4. Consider the decision problem "given a pair of non-negative integers p and q, is q divisible by p?". Formulate the problem according to the decision problem formalism. (Pick a suitable alphabet, a coding for pairs of integers etc.) What is the formal language corresponding to the problem?

5&6. A string of parantheses (that is, a string consisting solely of symbols "(" and ")") is balanced, if it is empty, it is a balanced string of parantheses enclosed in parantheses, or it consists of two concatenated balanced strings of parantheses. For example, ((()))() is a balanced string of parantheses but ()))(() is not.

   (a) Formulate the problem of recognizing balanced strings of parantheses as a decision problem. Describe also the corresponding formal language L_{bal}.
   (b) Write a program (in pseudocode) that solves the recognition problem corresponding to L_{bal}.

   You get credits for exercise 5 if you have completed part (a) and tried to do part (b). To get credits from exercise 6 also, you have to be ready to present a solution for both (a) and (b).

Note: There will be a weekly exercise session for English-speakers at Fridays starting at 8 am. The place is yet to be decided. See course www-page for the information.

If you need to have the exercises in English and/or plan to attend the English exercise sessions, please notify me by email (`matti.luukkainen@cs.helsinki.fi`).