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581330-2 Models for Programming and Computing

Exercise 4 (14.-18.2.2005)

1. Using pumping lemma show that the following language is not regular: $\{0^n 1^k 2^k \mid n, k \geq 0\}$
2. Using pumping lemma show that the following language is not regular: $\{ww \mid w \in \{a, b\}^*\}$
3. Let w^R = the string w written from right to left. (i.e. if $w = a_1 a_2 \ldots a_n$, then $w^R = a_n \ldots a_2 a_1$). A string w is a palindrome, if $w = w^R$. Let PAL be the language of palindromes of the alphabet $\{ a, b, c \}$, that is, PAL $= \{ w \in \{ a, b, c \}^* \mid w = w^R \}$.
   (a) Show that PAL is not regular.
   (b) Give a context-free grammar for PAL.
4. Let $L = \{x \in \{a, b\}^* \mid x$ contains twice as many as than bs $\}$.
   (a) Show that L is not regular.
   (b) Give a context-free grammar for L.
5. Give a context-free grammar using which you can generate the polynomes of a single variable x. You can assume that the terms of the polynome are in any order and that there can be several terms with the same degree. Eg. the following are legal polynomes that should be producible with your grammar: $2x\hat{}2 - 2x + 1$, $x - x\hat{}9 - 7 + 3x + 42$, $4x\hat{}0 + 10$.
6. Show that any class of regular languages over the alphabet Σ is closed under complementation, union and intersection, i.e., if languages $A, B \subseteq \Sigma^*$ are regular, so are also the languages $\overline{A} = \Sigma^* \setminus A$, $A \cup B$ and $A \cap B$. (Hint: For the complement, consider the deterministic finite automaton accepting A, and for the intersection, apply De Morgan's laws. — Comment: This result shows that we could also include complementation and intersection of regular expressions, without changing the corresponding class of languages.)