

Data structures, exercise 5, 16.-20.2.

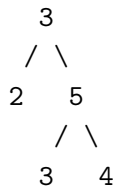
1. Insert to an empty binary search tree keys 2, 0, 3, 7, 9, 1, 5, 6 ja 8. Show how the tree grows when the insertions are made.

After the insertions, show how the keys 3, 9, 2 and 5 are deleted from the resulting tree.

2. Define an algorithm that counts the number of nodes in a binary tree.

What is the time and space complexity of the algorithm?

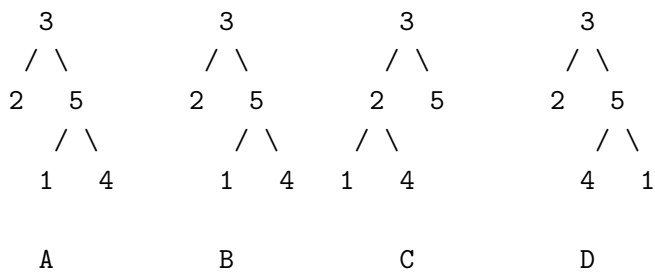
3. Define an algorithm that counts how many times a given key is in a binary tree. Eg. in the below tree, if the key to count is 3, the answer would be 2.



What is the time and space complexity of the algorithm?

4. Define an algorithm that checks if two binary search trees are same.

A and B below are same. A and B are not since the structures differ. A and D are not same since the keys differ.



5. Define an algorithm that prints keys in level order. That is, the keys at level i are printed before the keys at level $i + 1$ are printed.

What is the time and space complexity of the algorithm?

6. 2-3-tree is a type of balanced search tree.

A tree is 2-3-tree if the following are true:

- Every non leaf node has 2 or 3 children
- All the leaf nodes have same depth

- (a) Draw the greatest and smallest 2-3-tree with height 1, 2 and 3

- (b) Assume that 2-3-tree has height h . Show what is the smaller and upper limit of the number of the nodes.
- (c) Assume that 2-3-tree has n nodes. Show what is the smaller and upper limit of the height of the tree.

The following formula might be useful: $\sum_{i=0}^k a^i = \frac{a^{k+1}-1}{a-1}$ where $a \neq 1$.