LZ77 Parsing, et c.

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Seminar on Data Compression Techniques
LZ77 is a lossless compression method discovered by Abraham Lempel and Jacob Ziv in 1977
Lossless
(We can get all the data back)

vs.

Lossy
(We can’t necessarily)
Today

- LZ77 parsing (compression)

- Getting the data back (decompression)

- Variations on the basic scheme
  - Real compressors that use LZ
  - Rightmost parsing
  - Explicit literals
  - Relative pointers
  - Encoding the output of the parsing efficiently…
Lempel-Ziv (LZ) Parsing

The Lempel-Ziv parsing breaks a string $X$ of $n$ symbols into $z$ phrases.

If the parsing is up to position $i$, then the next phrase is either
- $X[i]$ – if symbol $X[i]$ has not appeared before, or
- $X[i..j]$ – the longest substring starting at $i$ and some $p_i < i$ in $X$

```
1 2 3 4 5 6 7 8 9 10 11
a b a a a b a b a b b b a
```
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a | b |
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The Lempel-Ziv parsing breaks a string X of n symbols into z phrases.

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- $X[i]$ – if symbol $X[i]$ has not appeared before, or
- $X[i..j]$ – the longest substring starting at i and some $p_i < i$ in X

```
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a b a a a b a b a b b a
a b a a b b
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```
1  2  3  4  5  6  7  8  9  10  11
a  b  a  a  b  a  b  a  b  b  a
a  b  a  a  b  a  b  a  b  a  b
(1,1) (1,3) (5,3) (2,2)
```
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a b a a a b a b a b b a
a | b | a | a b a | b a b | b a |

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$(1,1)$  $(1,3)$  $(5,3)$  $(2,2)$

**source**  **length**
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(a,0) (b,0) (1,1) (1,3) (5,3) (2,2)

**Literal phrases**

**Repeat phrases**
Applications of LZ77

Major technique of lossless data compression for almost 40 years
- gzip, 7zip, LZ4 are some popular file compressors
- Powerful compression

Also a very handy algorithmic tool for string processing
- Key to algorithms and data structures for repetition detection, covering, approximate pattern matching, compressed text indexing, et c., et c.

Lots of research on how to compute the parsing efficiently…
Näive parsing algorithm just scans the already parsed portion of the string, looking for matches...

ababbaabababazababbbzabbbz
Näive parsing algorithm just scans the already parsed portion of the string, looking for matches…

Computing the parsing

ababbaabababazababbzabbbz
Näive parsing algorithm just scans the already parsed portion of the string, looking for matches…

ababbaabababazababbzabbbzb
Naïve parsing algorithm just scans the already parsed portion of the string, looking for matches…

ababbaabababazababbzabbbz
Näive parsing algorithm just scans the already parsed portion of the string, looking for matches…

\[\text{aba} \text{bbaababa} \text{ba} \text{babazab} \text{abbzbab} \text{bbz}\]
Näive parsing algorithm just scans the already parsed portion of the string, looking for matches…

ababbaabababazababbzabb
Näive parsing algorithm just scans the already parsed portion of the string, looking for matches…

ababbaabababa
azababbzabbbz
Naïve parsing algorithm just scans the already parsed portion of the string, looking for matches…

```
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```
Näive parsing algorithm just scans the already parsed portion of the string, looking for matches…

Computing the parsing

ababbaabababazababbbzabbbz
Näive parsing algorithm just scans the already parsed portion of the string, looking for matches...

...something like $O(n^2)$ time
Most compressors maintain some data structure over the already parsed portion of the string to speed up matching.

\[
\text{ababbaabababazababbzabbbz}
\]
Faster parsing

Most compressors maintain some data structure over the already parsed portion of the string to speed up matching
- e.g. a hash table containing the locations of short matches

ababbaabababazababbbzabbbz
Most compressors maintain some data structure over the already parsed portion of the string to speed up matching.

aa: 6
ab: 1,3,7,9,11,15,17,21
ba: 2,5,8,10,12,16
bb: 4,18

ababbaabababazababbzabbbz
Faster parsing

Most compressors maintain some data structure over the already parsed portion of the string to speed up matching

aa: 6
ab: 1,3,7,9,11,15,17,21
ba: 2,5,8,10,12,16
bb: 4,18
...

ababbaabababazababbzabbbz
Getting the data back...
(i.e. decompression)
LZ Decoding

Decompressing the original file from the LZ phrases is simple & fast
– $O(n)$ time, $O(z)$ accesses to the already decoded part of the file
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(a,0),(b,0),(1,2),(2,2),(1,4),(1,3),(z,0),(1,5),(14,3),(4,2),(19,3),(14,2)
(26,4)(22,3)(1,3)(11,5)(13,4),(30,6)(1,1)
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ababbaababababa
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ababbaababababaz
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$(a,0), (b,0), (1,2), (2,2), (1,4), (1,3), (z,0), (1,5), (14,3), (4,2), (19,3), (14,2), (26,4), (22,3), (1,3), (11,5), (13,4), (30,6), (1,1)$

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ababbaabababazababbb
Decompressing the original file from the LZ phrases is simple & fast
- \(O(n)\) time, \(O(z)\) accesses to the already decoded part of the file

\[(a,0),(b,0),(1,2),(2,2),(1,4),(1,3),(z,0),(1,5),(14,3),(4,2),(19,3),(14,2),(26,4),(22,3),(1,3),(11,5),(13,4),(30,6),(1,1)\]

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ababbaabababazababbzab
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$\Rightarrow$

$\text{ababbaabababazababbzab}$
Decompressing the original file from the LZ phrases is simple & fast
- $O(n)$ time, $O(z)$ accesses to the already decoded part of the file

```
(a,0), (b,0), (1,2), (2,2), (1,4), (1,3), (z,0), (1,5), (14,3), (4,2), (19,3), (14,2)
(26,4), (22,3), (1,3), (11,5), (13,4), (30,6), (1,1)
```

ababbaaabababazababbbzabb
c = 0
for each phrase \((p,l)\)
    if \(l == 0\) then /*literal phrase*/
        \(S[c++] = p\)
    else /*repeat phrase*/
        for \(i = 0\) to \(l-1\) do
            \(S[c++] = S[p++]\)
Dealing with big files...
Practicalities with big files

• Big files cause difficulties for LZ77, for at least two reasons:
  
  – During compression: the index data structures used to perform parsing efficiently grow linearly with the input size – on big files they can start to exceed this size of memory
  
  – During decompression: copying characters from the already decompressed part of the file requires a random access to retrieve those characters. If the decompressed part of the file exceeds memory, this random access will become a disk access (which is very slow)
  
  – Real compressors take shortcuts to deal with these situations…
Workaround (i.e. hack) for big inputs

At compression time, break file into blocks, compress each block separately, source & phrase must be in same block
Workaround (i.e. hack) for big inputs

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At compression time, break file into blocks, compress each block separately, source & phrase must be in same block

(...)(...)(...)(...)
At compression time, break file into blocks, compress each block separately, source & phrase must be in same block

(.,.)(.,.)(.,.)(.,.)(.,.)(.,.)
At compression time, break file into blocks, compress each block separately, source & phrase must be in same block.
A fixed-length window slides over the text as we parse
  – Source for the next phrase must be inside the window

(.,.)(.,.)(.,.)…

(.,.)…
Similar idea: use a sliding window

A fixed-length window slides over the text as we parse
  – Source for the next phrase must be inside the window

\[ (\ldots)(\ldots)(\ldots) \ldots \]
A fixed-length windows slides over the text as we parse
   - Source for the next phrase must be inside the window

(,,)(,,)(,,)... (,,)...
A fixed-length window slides over the text as we parse

- Source for the next phrase must be inside the window

(...)(..)(..)...

(...)

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Workaround (i.e. hack) for big inputs

No free lunch...

![Bar chart showing the number of phrases in billions for Wikipedia, DNA reads, and Source code. The chart compares the whole file and 4GB blocks versions.](image)
Workaround (i.e. hack) for big inputs

No free lunch…

Bar chart showing the number of phrases (billions) for Wikipedia, DNA reads, and source code, comparing whole file to 4GB blocks, with a >10x difference in DNA reads.
Variations to the basic scheme...
Variations

- The greedy parsing algorithm minimizes # of phrases…

- …but often we are more interested in minimizing the total size of the compressed file.

- This forces us to think about how (pos,len) pairs are encoded,
  - and whether we should use the (pos,len) representation of a phrase at all.
Variations: encode short strings as literals

- Naive encoding: store \((\text{pos}, \text{len})\) as two 32-bit integers
  - This equates to 8 bytes per phrase
  - If we assume each character of the input needs 1 byte...
  - ...it’s only worth encoding a phrase as \((\text{pos}, \text{len})\) if its “fairly” long
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… (14,3), (4,12), (19,3), (14,26), (26,18), (22,2)…
Variations: encode short strings as literals

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  - ...it’s only worth encoding a phrase as \((pos, len)\) if its “fairly” long

... \((14, 3), (4, 12), (19, 3), (14, 26), (5, 18), (22, 2)\)...

... \((-3, \text{ktt}), (12, 4), (-3, \text{vav}), (26, 14), (18, 5), (-2, \text{nc})\)...
• Often in practice, a long match is broken by a single mismatching character
  – In the parsing we’ve discussed, this will produce three phrases

actgcagagcgcgcagagcctac ....... actgcagagcgcacagagccctat
Variations: encode triples rather than pairs

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actgcagagcgcgcagagcccta ....... actgcagagcgcacagagccctat

$(1, 13), (1, 2), (12, 10)$
Variations: encode triples rather than pairs

- Often in practice, a long match is broken by a single mismatching character
  - In the parsing we’ve discussed, this will produce three phrases

```
actgcagagcgcgagccctac ....... actgcagagcgcagagccctat
```

```
(1,13),(1, 2),(12,10)

(1,13,a), (12,10,t) ...
```
• Many more tricks used in actual compressors…

• E.g., store distance to previous match rather than position of previous match
  – Numbers tend to be smaller, so use less bits

• E.g., store positions relative to the previous phrase’s position
  – These tend to be correlated
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Whatever the output of the parsing algorithm (some stream of integers and chars) it needs to be coded efficiently…