Viimeksi saatiin:

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + f\bar{v} - \frac{\partial}{\partial z}\overline{u'w'}$$

$$\frac{\partial \bar{v}}{\partial t} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y} - f\bar{u} - \frac{\partial}{\partial z}\overline{v'w'}$$

$$\frac{\partial \bar{\theta}}{\partial t} = \overline{S_\theta} - \frac{\partial}{\partial z}\overline{\theta'w'}$$

$$\frac{\partial \bar{q}}{\partial t} = \overline{S_q} - \frac{\partial}{\partial z}\overline{q'w'}$$

Turbulenssitermit ovat uusia tuntemattomia

Edes keskiarvoja ei voi ennustaa ilman

Palataan hieman taaksepäin

$$\frac{\partial s}{\partial t} + V \cdot \nabla s = F$$ ← sisältää kaikenlaista

$$\frac{\partial \bar{s}}{\partial t} + \frac{\partial s'}{\partial t} + \bar{V} \cdot \nabla \bar{s} + \bar{V} \cdot \nabla s' + V' \cdot \nabla \bar{s}' + V' \cdot \nabla s' = \bar{F} + F'$$ ← A

Reynolds-keskiarvoistamalla saatiin

$$\frac{\partial \bar{s}}{\partial t} + \bar{V} \cdot \nabla \bar{s} = -\nabla \cdot \overline{s'V'} + \bar{F}$$ ← B

Mutta voidaan myös laskea

$$A - B \Rightarrow \frac{\partial s'}{\partial t} = (\text{jotain})$$

Esimerkiksi u', v', w' kanssa voidaan kikkailla:

$$\frac{\partial u'}{\partial t} = (\ldots)$$

$$2u' \frac{\partial u'}{\partial t} = 2u'(\ldots)$$

$$\frac{\partial}{\partial t}(u')^2 = 2u'(\ldots)$$, keskiarvoistetaan

$$\frac{\partial}{\partial t} \overline{u'^2} = (\text{jotain})$$

Samalla idealla:

$$\begin{cases} \dfrac{\partial u'}{\partial t} = (\ldots) \\ \\ \dfrac{\partial v'}{\partial t} = (\ldots) \end{cases} \Rightarrow \begin{cases} v'\dfrac{\partial u'}{\partial t} = v'(\ldots) \\ \\ u'\dfrac{\partial v'}{\partial t} = u'(\ldots) \end{cases}$$

$$\left(\frac{\partial v'u'}{\partial t} = v'\frac{\partial u'}{\partial t} + u'\frac{\partial v'}{\partial t} \right)$$

$$\frac{\partial}{\partial t}\overline{v'u'} = (\text{jotain})$$

$$\frac{\partial}{\partial t}\overline{u'^2} = (\text{jotain})$$

$$\frac{\partial}{\partial t}\overline{u'v'} = (\text{jotain})$$

Näissä aina "jotain" sisältää termejä muotoa $\overline{u'^3}$, $\overline{u'^2v'}$, $\overline{u'v'w'}$, jne.

ja ne ovat uusia tuntemattomia.

1. aste

$\overline{u}$

$\overline{v} \quad \overline{w}$

2. aste

$\overline{u'^2}$

$\overline{u'v'} \quad \overline{u'w'}$

$\overline{v'^2} \quad \overline{v'w'} \quad \overline{w'^2}$

3. aste

$\overline{u'^3}$

$\overline{u'^2v'} \quad \overline{u'^2w'}$

$\overline{u'v'^2} \quad \overline{u'v'w'} \quad \overline{u'w'^2}$

$\overline{v'^3} \quad \overline{v'^2w'} \quad \overline{v'w'^2} \quad \overline{w'^3}$

Samoin voidaan kirjoittaa myös termeille

$$\frac{\partial \overline{\theta'^2}}{\partial t} = (\ldots)$$

$$\frac{\partial \overline{\theta' w'}}{\partial t} = (\ldots) \qquad \text{jne.}$$

Tai 3. asteessa esim. $\frac{\partial}{\partial t} \overline{\theta'^2 w'} = (\ldots)$

Parametrisointi

"keksitään" joku kaava millä tuntematon termi "ennustetaan" tunnetuista

$$\overline{\theta' w'} = f(\bar{u}, \bar{v}, \bar{w}, \theta, q)$$

0 asteen parametrisointi

Ei ratkota ennustusyhtälöitä edes keskisuureille, vaan keksitään suoraan niitä kuvaavat funktiot.

$$\bar{u}(z,t) = f(z,t)$$

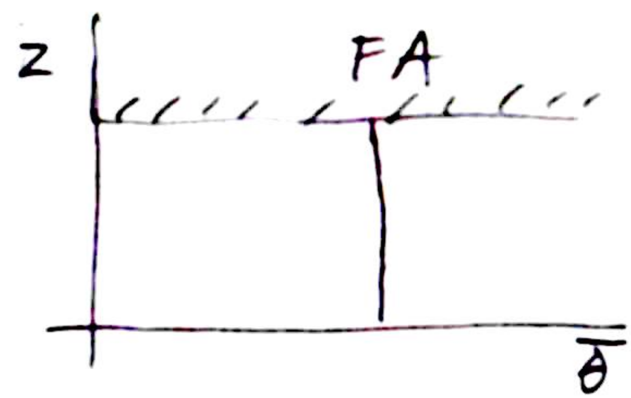
jne.

0.5 asteen parametrisointi

Ei keksitä koko ratkaisua, mutta profiilin muoto.

Esim. $\bar{\theta}(z) =$ vakio

$$\bar{\theta}(z,t) = \bar{\theta}(z, t=0) + \Delta\bar{\theta}(t)$$



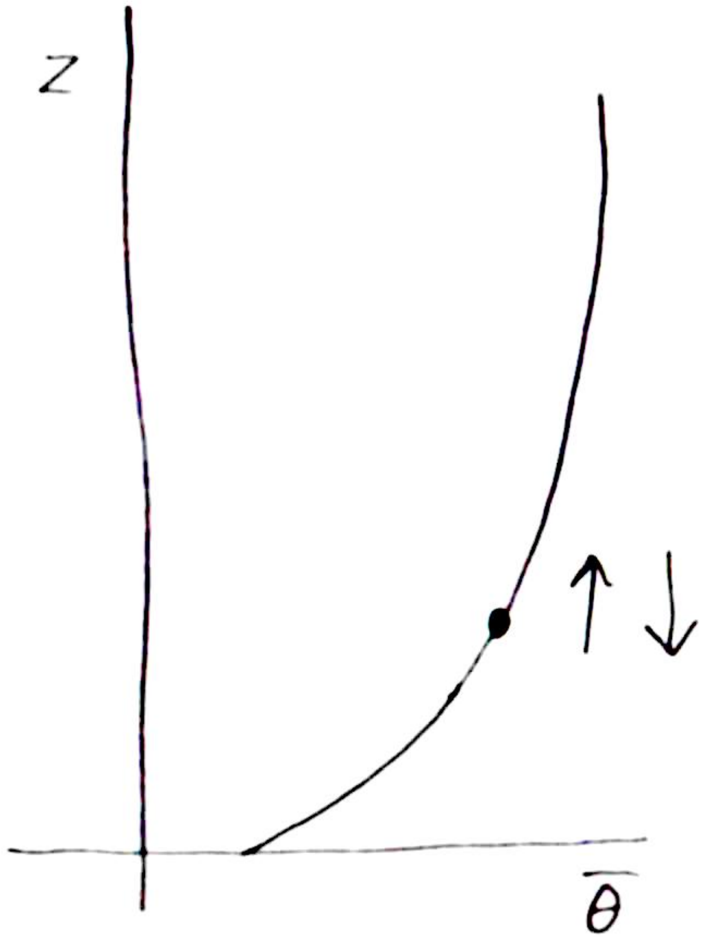
1. asteen parametrisointi

vaihtokerrointeoria, eli

K-teoria eli diffuusioanalogia

$$\overline{u'w'} = -K_m \frac{\partial \bar{u}}{\partial z} \qquad \overline{v'w'} = -K_m \frac{\partial \bar{v}}{\partial z}$$

$$\overline{\theta'w'} = -K_h \frac{\partial \bar{\theta}}{\partial z} \qquad \overline{q'w'} = -K_h \frac{\partial \bar{q}}{\partial z}$$



vuo = vakio
x
gradientti

diffusio

lämpöyhtälö

Nyt meillä on:

$$\frac{\partial}{\partial t}\bar{u} = f(\bar{v} - v_g) + \frac{\partial}{\partial z}\left(K_m \frac{\partial \bar{u}}{\partial z}\right)$$

$$\frac{\partial}{\partial t}\bar{v} = -f(\bar{u} - u_g) + \frac{\partial}{\partial z}\left(K_m \frac{\partial \bar{v}}{\partial z}\right)$$

$$\frac{\partial}{\partial t}\bar{\theta} = \overline{S_\theta} + \frac{\partial}{\partial z}\left(K_h \frac{\partial \bar{\theta}}{\partial z}\right)$$

$$\frac{\partial}{\partial t}\bar{q} = \overline{S_q} + \frac{\partial}{\partial z}\left(K_h \frac{\partial \bar{q}}{\partial z}\right)$$

Jos K olisi vakio, niin meillä olisi suoraan diffuusio/lämpöyhtälö:

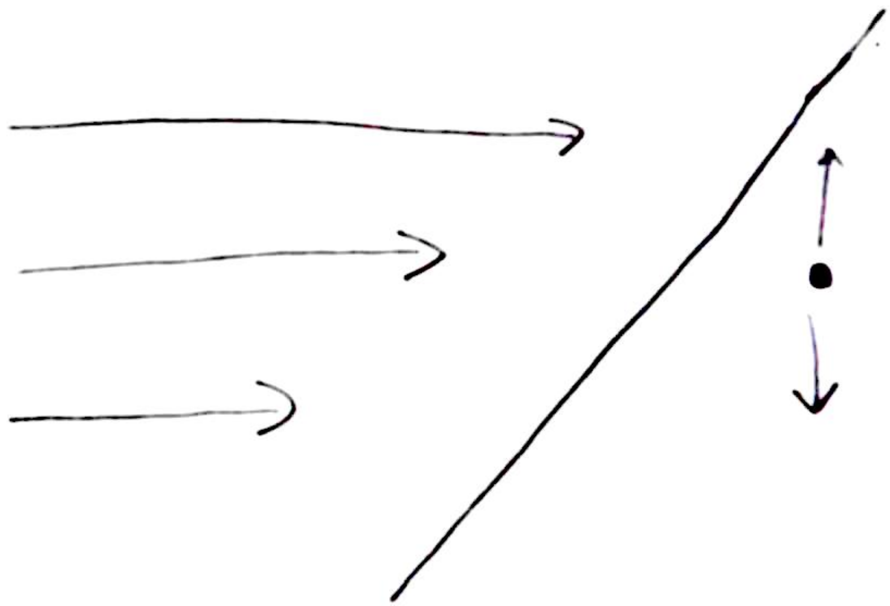
$$\frac{\partial \bar{\theta}}{\partial t} = \ldots + K_h \frac{\partial^2}{\partial z^2} \bar{\theta}$$

Vakio-K:t eivät kuitenkaan tuota kovin hyviä tuloksia.

K = eddy viscosity

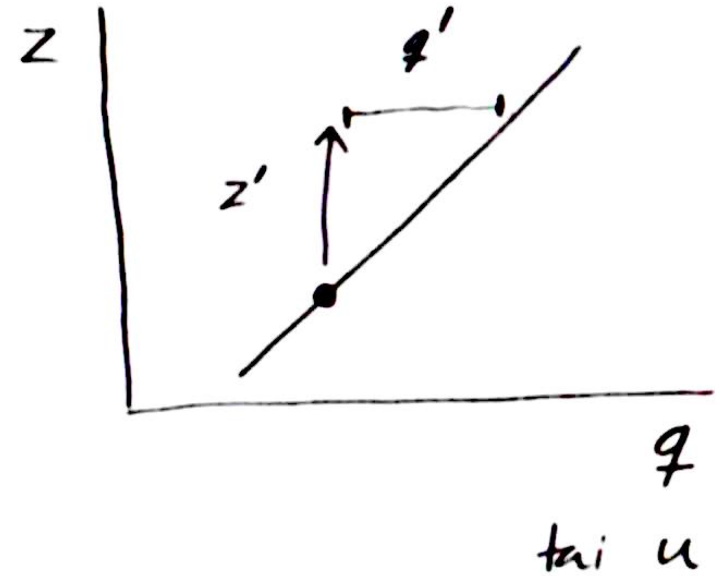
Sivuhuomautus:

(kaasun) viskositeetista



$\mu \sim v \lambda$

Sekoitusmatkateoria (kädet heiluu)



$$q' \sim -z' \frac{\partial \bar{q}}{\partial z}$$

$$u' \sim -z' \frac{\partial \bar{u}}{\partial z}$$

$$w' \sim |u'|$$

$$\overline{w'q'} \sim -\overline{z'^2} \left|\frac{\partial \bar{u}}{\partial z}\right| \frac{\partial \bar{q}}{\partial z}$$

$$\overline{w'q'} \sim -\overline{z'^2}\left|\frac{\partial \bar{u}}{\partial z}\right|\frac{\partial \bar{q}}{\partial z}$$

$$\overline{w'q'} = -K_h \frac{\partial \bar{q}}{\partial z}$$

$$\Rightarrow \quad K_h \sim \overline{z'^2}\left|\frac{\partial \bar{u}}{\partial z}\right|$$

Korkeammalla isommat pyörteet

sekoitusmatka $\quad l = k z$

$$K_h = l^2 \left|\frac{\partial \bar{u}}{\partial z}\right|$$

k = von Kármánin vakio
n. 0.40

(Prandl 1925)

Oletus $w' \sim |u'|$ vaatii isotrooppista turbulenssia.

Toimii jos turbulenssi syntyy mekaanisesti, eli neutraalissa stabiliteetissa.

Lisäksi oletettiin lineaariset

gradientit

$$\frac{\partial \bar{u}}{\partial z} \quad , \quad \frac{\partial \bar{q}}{\partial z}$$

eli toimii lyhyillä matkoilla/

pienessä skaalassa.

TKE turbulenssin kineettinen energia

Prujussa: k Stullin kirjassa: e

$$\overline{k} = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)$$

Oikeastaan liike-energia olisi

$k = \frac{1}{2}\rho\overline{V'^2}$, mutta ajatellaan että k on per "kilo ilmaa", joten ρ jää pois

Aiemmin puhuttiin miten $\overline{u'^2}$ $\overline{v'^2}$ $\overline{w'^2}$
voisi lasken (aika urakka olisi!)

Prujun s. 9 yhtälöissä ei edes ole
nostetermiä $g\frac{\theta'}{\overline{\theta}}$ mukana. (Eikä viskositeettia)

Aloittaen jostain prujua yleisemmästä,
voitaisiin kuitenkin lasken ...

(ks. Stull, luku 4, jos kiinnostaa)

TKE (tärkeä!)

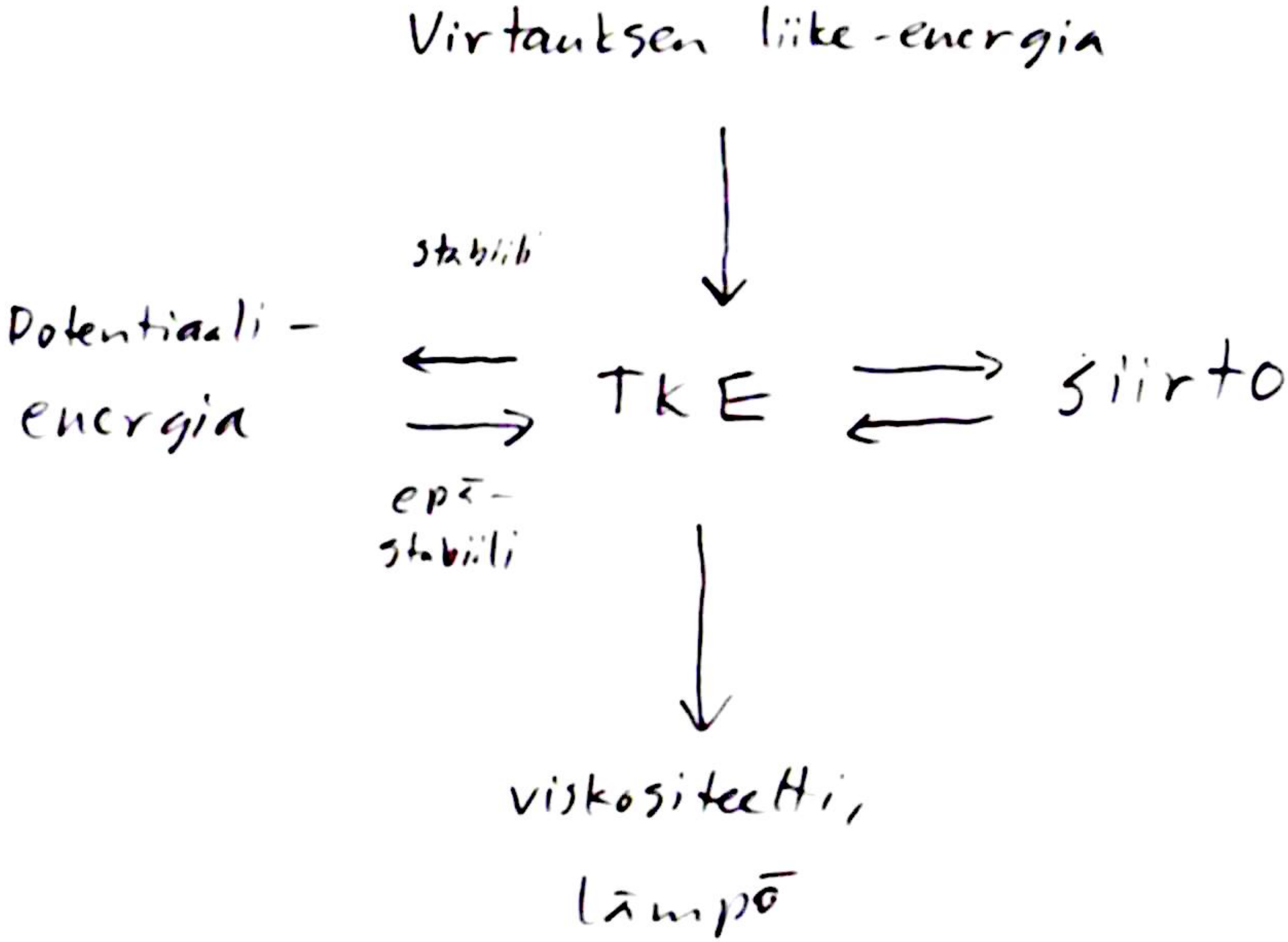
$$\frac{\partial k}{\partial t} = \underbrace{-\frac{\partial}{\partial z}\left[\overline{k_t w'} + \frac{\overline{p'w'}}{\bar{p}}\right]}_{T} \underbrace{-\overline{u'w'}\frac{\partial \bar{u}}{\partial z} - \overline{v'w'}\frac{\partial \bar{v}}{\partial z}}_{S} \underbrace{+\frac{g}{\bar{\theta}}\overline{w'\theta'}}_{B} \underbrace{-\varepsilon}_{D}$$

T = Transport

S = Shear, mekaaninen turbulenssin tuotto

B = Buoyancy, noste (voi myös vaimentaa)

D = Dissipaatio, kitka, viskositeetti


Virtauksen liike-energia
stabiili
Potentiaali-energia
TKE
siirto
epä-stabiili
viskositeetti, lämpö

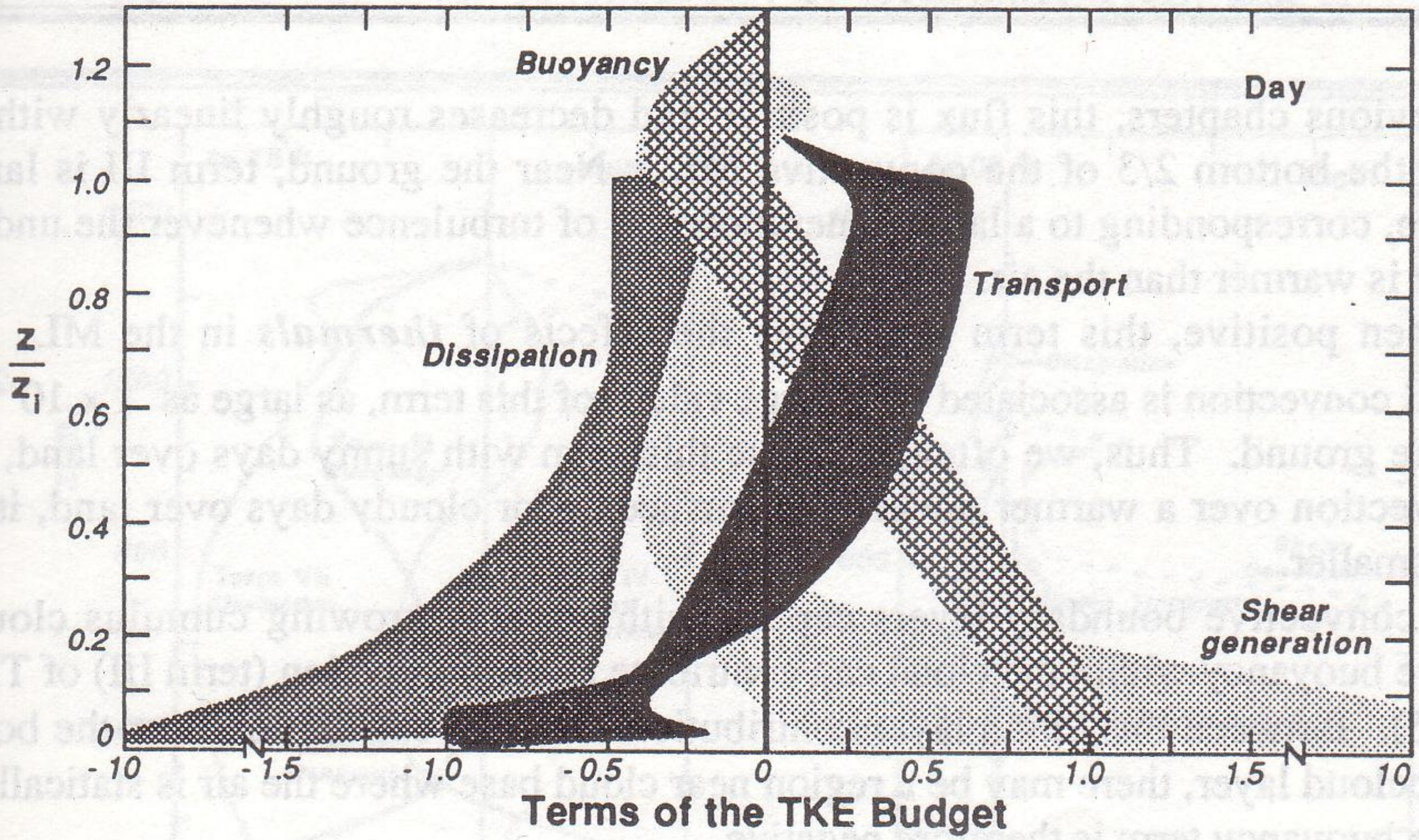


Fig. 5.4 Normalized terms in the turbulence kinetic energy equation. The shaded areas indicate ranges of values. All terms are divided by w_*^3 / z_i, which is on the order of $6x\ 10^{-3} m^2 s^{-3}$. Based on data and models from Deardorff (1974), André et al. (1978), Therry and Lacarrere (1983), Lenschow (1974), Pennell and LeMone (1974), Zhou, et al. (1985) and Chou, et al. (1986).

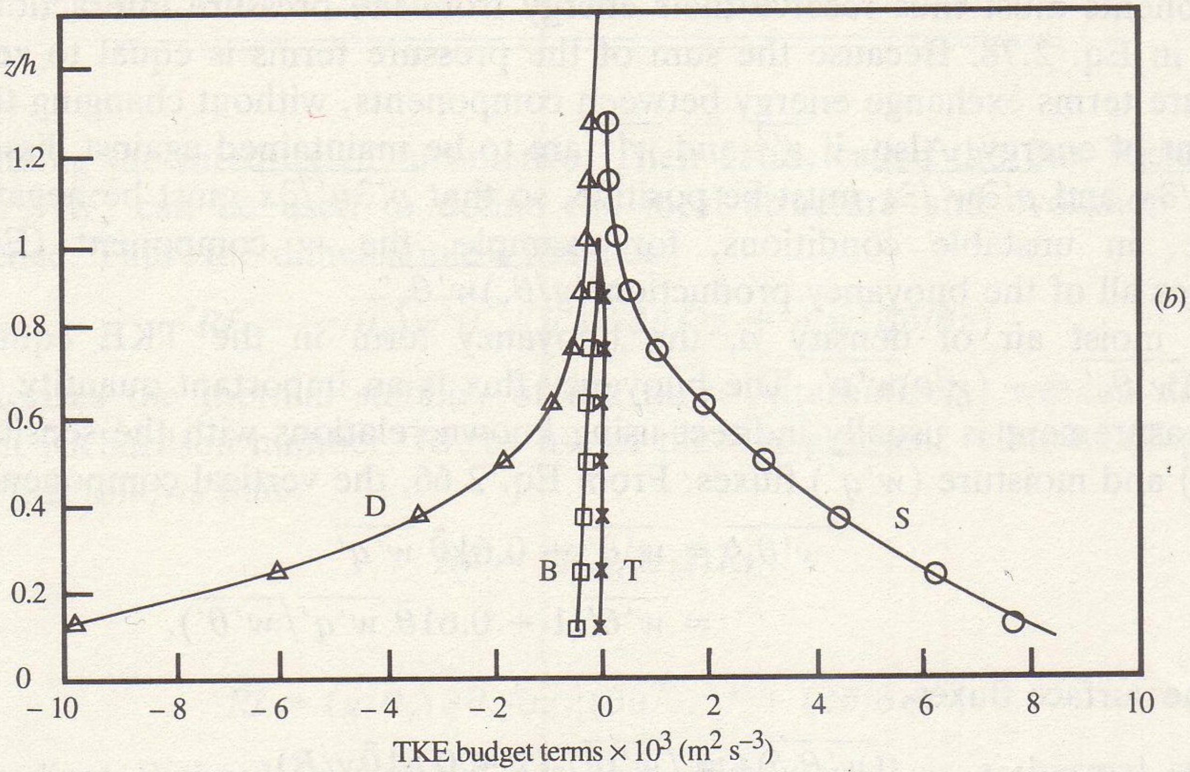


Fig. 2.4 Terms in the TKE equation (2.74b) as a function of height, normalized in the case of the clear daytime ABL (*a*) through division by $w_*{}^3/h$; actual terms are shown in (*b*) for the clear night-time ABL. Profiles in (*a*) are based on observations and model simulations as described in Stull (1988; Figure 5.4), and in (*b*) are from Lenschow *et al.* (1988) based on one aircraft flight. In both, B is the buoyancy term, D is dissipation, S is shear generation and T is the transport term. Reprinted by permission of Kluwer Academic Publishers.

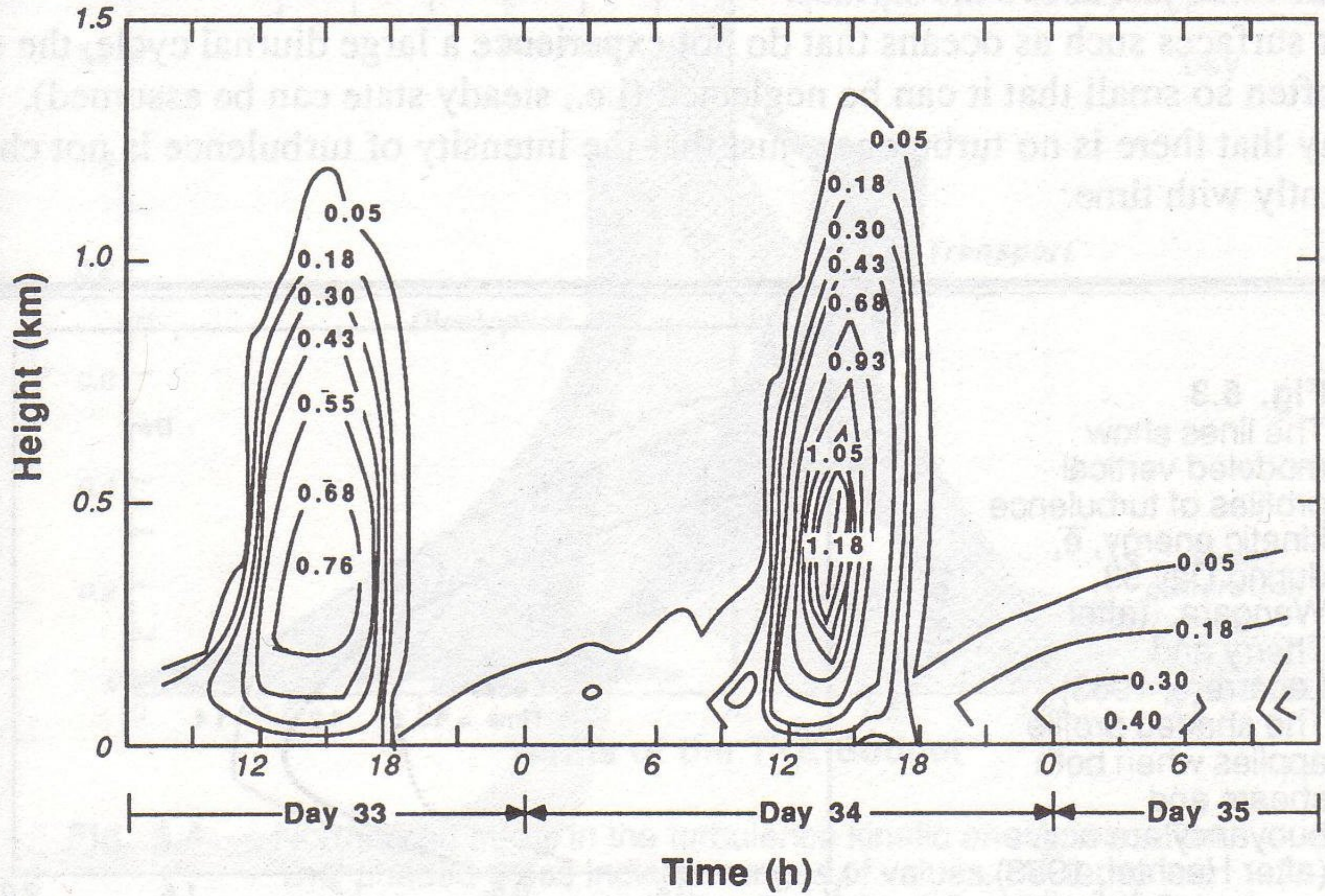


Fig. 5.1 Modeled time and space variation of $\bar{e}$ (turbulence kinetic energy, units m^2s^{-2}), for Wangara. From Yamada and Mellor (1975).

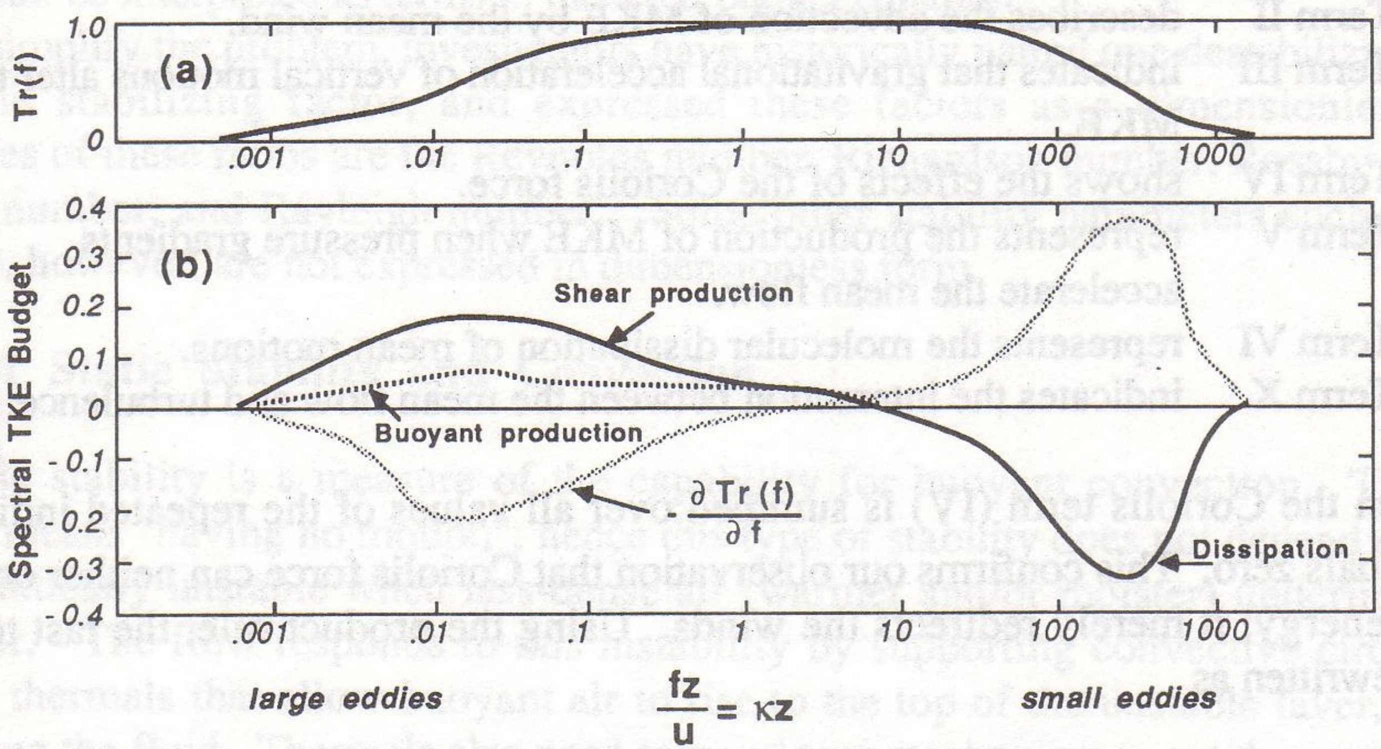


Fig. 5.16 Example of spectral energy budget terms for z/L = -0.29. Shown in (b) are the shear and buoyant production and the dissipation terms as functions of frequency f. The shaded curve (labelled $\partial Tr(f) / \partial f$) is equal to minus the sum of the shear, buoyant production and dissipation terms. The Tr(f) curve (a) was obtained by integrating $\partial Tr(f) / \partial f$. Here Tr(f) is the transfer of energy in f space required to balance the production and dissipation. The symbol f is frequency and κ is wave number. After McBean and Elliott (1975).

1.5 asteen parametrisointi

$$\bar{u} \qquad \overline{u'^2}$$

$$\bar{v} \quad \bar{w} \qquad \overline{u'v'} \quad \overline{u'w'}$$

$$\overline{v'^2} \quad \overline{v'w'} \quad \overline{w'^2}$$

$$TKE = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)$$

eli oletetaan $\overline{u'^2} = \overline{v'^2} = \overline{w'^2}$

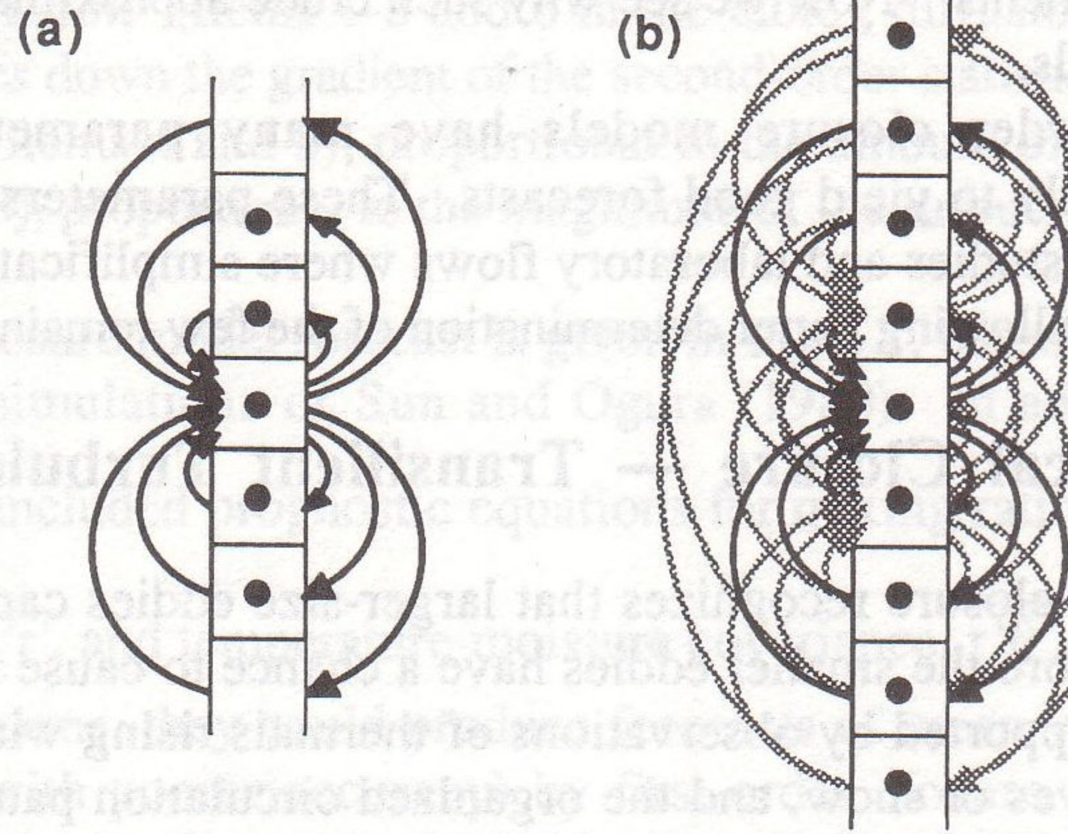


Fig. 6.8
(a) Schematic idealization of the eddies that mix air to and from the center grid box, in a 1-D column of air. (b) Superposition of eddies acting on 3 of the grid boxes. After Stull (1984).