Advanced Data Structures (Spring 2007)

Exercise 2 (Wed 28.3., 12-14, C221)

1. **Open addressing.**
   Recall/find out what is hashing with *open addressing.*
   
   (a) What types of open addressing are there?
   
   (b) How does open addressing compare with chaining?

2. **Rank/select dictionary.**
   A rank/select dictionary is a dictionary-like data structure for an ordered set $S \subseteq U$ that supports the operations:
   
   - **rank**($x$): the number of keys in $S$ that are smaller or equal to $x$.
   - **select**($i$): the $i$th key in $S$.

   (a) How can neighbor queries be implemented using rank and select?
   
   (b) Develop a *dynamic rank/select dictionary* based on balanced search tree with $O(n)$ space ($O(n \log u)$ bits) and $O(\log n)$ time operations.

3. **Ordered minimal perfect hashing.**
   Read the definition of *ordered minimal perfect hash function* in the perfect hashing survey, page 4 (the material for Monday’s study group).

   (a) Is rank($x$) $- $1 (see Problem 2) an ordered minimal perfect hash function?
   
   (b) If $h$ is an arbitrary ordered minimal perfect hash function, how can it differ from rank($x$) $- $1? Can $h$ be used for implementing neighbor queries instead of rank?

4. **Linear congruential functions.**
   The linear congruential functions:

   $$\mathcal{H}_p = \{ h_a : x \mapsto (ax \mod p) \mod m \mid a \in \{0, 1, \ldots, p-1\} \}$$

   form a universal family of hash functions provided that $p$ is prime larger or equal to $u$. Show with an example what can go wrong if

   (a) $p$ is not a prime.
   
   (b) $p$ is (much) smaller than $u$ (but still larger than $m$).