582206 Models of Computation (Autumn 2009)
1st Exam, 22 October at 9-12
Lecturer: Juha Kärkkäinen

For ease of grading, please write your answer to each of the problems 1, 2 and 3 on its own sheet of paper. Return an answer sheet for each question even if you leave the answer empty. Please write on each sheet: your name, student number or identity number, signature, course name, exam date and the problem number.

1. [*4+3+3 points*] In all parts, the alphabet is $\{a, b\}$.

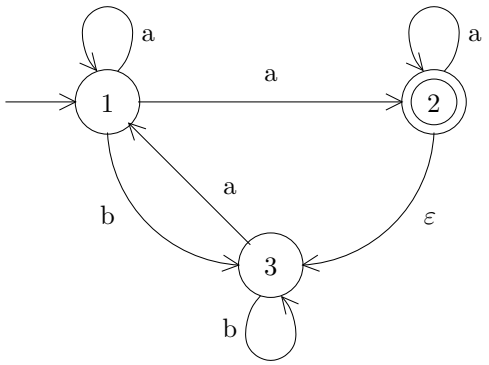
   (a) The language A consists of strings at least two symbols long such that the first symbol is the same as the last symbol. Give *both* a (deterministic or nondeterministic) finite automaton that recognizes the language A *and* a regular expression the represents the language A.

   (b) Transform the regular expression

   $$(a \cup bb)^* b$$

   into an equivalent nondeterministic finite automaton using the construction described on the course.

   (c) Transform the nondeterministic finite automaton given below into an equivalent deterministic finite automaton using the construction described on the course.



   In parts (a) and (b), you do not need to describe the construction or give any intermediate steps as long as the final answer is clearly a result of the proper construction.

2. [*4 points*] Describe how to tranform an arbitrary nondeterministic finite automaton into an equivalent automaton that has a single accept state. Give both a short informal description with pictures and a precise mathematical construction based on the formal definition of a nondeterministic finite automaton.

3. [*4+2+4 points*]

   (a) Which of the following languages are regular and which are not?

   $$A_1 = \{ a^n b^m a^n \mid m, n \in \mathbb{N} \} \qquad A_3 = \{ a^n a^n b^m \mid m, n \in \mathbb{N} \}$$
   $$A_2 = \left\{ a^k b^m a^n \mid k, m, n \in \mathbb{N} \right\} \qquad A_4 = \{ a^n a^m b^n \mid m, n \in \mathbb{N} \}$$

   No proof or explanation is necessary.

   (b) Choose one of the regular languages in part (a) and show that it is regular.

   (c) Choose one of the non-regular languages in part (a) and show that it is not regular.

   In parts (b) and (c), you can use any general results presented on the course, but not results that directly say that a specific language is regular or non-regular.