582206 Models of Computation (Autumn 2009)
Exercise 2 (15–18 September)

Basic exercises

The first three problems are basic applications of the material from the text book. Solve them by yourself; if there is anything unclear you can ask about it during the exercise session.

1. Give a verbal description for the following sets:

    (a) $\{\, 2n + 1 \mid n \in \mathbb{N} \,\}$

    (b) $\{\, ww^{\mathcal{R}} \mid w \in \{\, 0, 1 \,\}^{*} \,\}$

    (c) $\{\, u \in \Sigma^{*} \mid$ for some $v \in \Sigma^{*}$ we have $uv = \texttt{abrakadabra} \,\}$, where $\Sigma = \{\, \texttt{a}, \ldots, \texttt{z} \,\}$.

    (On this course we include zero as a natural number, *i.e.*, $\mathbb{N} = \{\, 0, 1, 2, \ldots \,\}$.)

2. Write formal definitions for the following sets:

    (a) palindromes over the alphabet $\{\, \mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d} \,\}$

    (b) natural numbers evenly divisible by 3

    (c) stings over alphabet $\{\, 0, 1 \,\}$ where all the 0s precede any 1s.

    (A *palindrome* is a string that reads the same forwards and backwards.)

3. (a) Give a formal definition for the following finite automaton. What is the sequence of states the automaton enters on the following inputs: 0101, 1010 and 111000. What is the language recognised by the automaton?
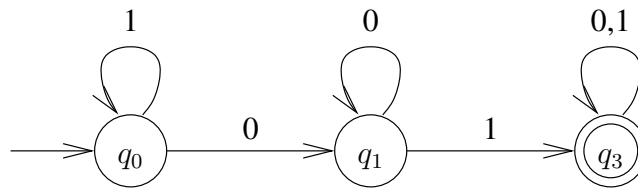
    (b) Let $M = (\{\, q_0, q_1, q_2 \,\}, \{\, 0, 1 \,\}, \delta, q_0, \{\, q_1 \,\})$, where δ is as follows:

	0	1
q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_1	q_2

    Draw the automaton as a state diagram. What is the sequence of states the automaton enters on the following inputs: 01010, 1010 and 000111. What is the language recognised by the automaton?

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Discussion problems

Read the following problems and make sure you are familiar with the necessary basic concepts. You are not expected to solve the problems by yourself; we shall discuss them together.

4. [Sipser Problem 1.51] Let L be any language. We say that strings x and Y are *distinguishable by L* if a string z exists such that exactly one of xz and yz is in L. Otherwise x and y are *indistinguishable* by L and we write $x \equiv_L y$. Show that $\equiv_L$ is an equivalence relation.

5. Consider the task of designing a control system for an elevator in an office building.

   Each floor has a button for summoning the elevator, and inside the elevator there are buttons for each floor as usual.

   The control system gets as input all the pressed buttons and additionally a signal whenever the elevator enter a new floor. The control system should decide whether at any given time the elevator should be going up or down, or remain where it is.

   For simplicity, we assume some other mechanism causes the elevator to pause in each floor for the necessary time, opens and closes the doors, etc. We also assume that no two buttons are ever pressed simultaneously.

   Design a control algorithm that guarantees that anybody needing the elevator will get to his destination eventually. How would you apply the idea of a finite automaton in such a system?

6. Prove that $2^n \geq n^2$ holds for all $n \geq 4$.