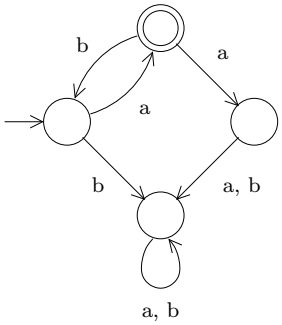
582206 Models of Computation (Autumn 2009)
Exercise 3 (22–25 September)

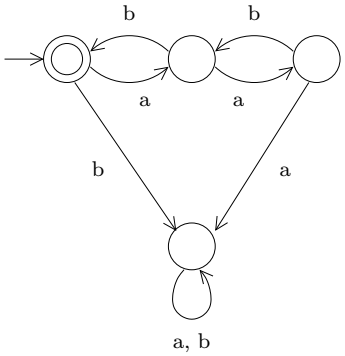
Here and later when you are asked to give an automaton, you should give it as a state diagram (as in Problem 2(b) below), unless otherwise stated.

1. For any language A, define
$$A^{\mathcal{R}} = \{ w^{\mathcal{R}} \mid w \in A \}$$
where $w^{\mathcal{R}}$ is the reverse of w, i.e., if $w = w_1, \ldots, w_n$ then $w^{\mathcal{R}} = w_n, \ldots, w_1$. Show that $A^{\mathcal{R}}$ is regular if $A = L(M)$ for some finite automaton M with at most one accept state.

2. (a) Let $A = \{ \mathrm{a} \}$ and $B = \{ \mathrm{b}, \mathrm{c} \}$. List the elements of the set $(A \circ B)^*$, which are no longer than 5.

   (b) Describe in English the languages recognized by the following automata:





3. Give a finite automaton recognizing the following languages of the alphabet $\{ \mathrm{a}, \mathrm{b} \}$:

   (a) strings that contain exactly one b

   (b) strings where every odd-numbered symbol is b

   (c) the union of the languages in (a) and (b).

   In (c), use the union automaton construction (Sipser, Theorem 1.25).

4. How many states is needed in a finite automaton recognizing the language $L = \{ 0^k 1^k \mid 0 \leq k \leq n \}$? Justify your answer.